Normative Sentences and Logic

This final thesis (Danish: *speciale*) is about *normative* logics. Normative logics are applied to normative sentences or to the semantic content of normative sentences. *Normative sentences* express obligations, permissions and prohibitions. In some sense obligations, permissions and prohibitions are the contents of normative sentences but the ontological status of these categories is at least as controversial as the status of propositions, usually taken as contents of declarative sentences. More about this can be found in Part II (A) of this thesis. The category of normative sentences includes indicative sentences containing normative verbs, adjectives, or adverbs, such as "He must run", or "You are allowed to run" or "They are prohibited from running".[1] These are usually called deontic sentences. It also includes imperatives such as "Run!". Normative sentences are either deontic sentences or imperatives. Deontic sentences have more expressive power than imperatives. Imperatives usually express second person obligations and prohibitions, whereas deontic sentences can express all obligations, prohibitions and permissions.[2] A very controversial question regards the relationship between imperatives and deontic sentences.[3] Are they essentially the same or essentially different?[4]

Logic can be described as the study of inferences. From a set of sentences we infer another. Normative logic is the study of inferences including normative sentences. An example of a logical inference including imperatives is the following. The sentences above the line are called the premises; the sentence below the line is called the conclusion.

If you see an eagle, then lift up your head!
You see an eagle.
____
Lift up your head!

The first premise is a mixed conditional, consisting of a declarative sentence (the antecedent) and an imperative (the consequent). The second premise is a declarative sentence. The conclusion is an imperative. With regard to normative logic there are problems of devising a logic, problems of formalising ordinary normative language and problems concerning the meaning of ordinary

[1] The modal verbs may express deontic, epistemic or alethic modalities depending on the context. "He may run now", for example, can mean that he is permitted to run now(deontic modality) or that there is a certain probability that he is running now (epistemic modality) depending on intonation, situation, and so on.

[2] In general we do not express permissions with imperatives. An unusual exception, pointed out in (Hamblin, 1987) p. 29-30, are permissions expressed by a conditional such as "If you feel like it, go ahead!" where the consequent is an imperative. Sometimes sentences with "let" as in "let it rain!" are regarded as third person imperatives.

[3] Cf. (Hamblin, 1987), p. 18.

[4] About imperatives as a syntactical category and deontic sentences as a semantic category see part II.

normative sentences, e.g. of ascribing a semantic predicate such as *truth* to normative sentences. With the somewhat bad reputation normative logics have fallen into mainly because of the deontic paradoxes, it is important to confront the more philosophical questions of meaning. It will not do to simply introduce technical devices and start doing logic, as was perhaps once hoped.

Normative logics include deontic logics and imperative logics. Part I of this thesis gives a historical overview over normative logics. In Part II (B) a new logical system for imperatives (mandates) is proposed based on the philosophical foundation of a theory of meaning for imperatives (A).

PART I

A HISTORICAL OVERVIEW OF NORMATIVE LOGICS FROM ERNST MALLY TO DEFEASIBLE DEONTIC LOGIC

One may distinguish normatives or prescriptive language (deontic sentences, imperatives) from descriptive language (declarative sentences) and from evaluative language (value judgements). It has been argued that practical reason is ruled by different logical principles than theoretical reason, separating deontic logic from classical propositional logic.[5] (A system of classical propositional logic is sound and complete with respect to bivalent truth functional semantics). Normative logics may also be separated from axiological logics, which deal with values and preferences expressed by evaluative language.[6]A specific trend of research combines deontic logic with axiological logic, creating deontic logic based on preference semantics.[7] Others follow grammarians who divide language into imperatives, indicatives, interrogatives, etc. We may need a logic of imperatives, a logic of questions and so forth.[8] This view possibly dates back to the Stoics.[9]
Still others, following the main trend of modal logic, focus on different ways of qualifying or propositions by adverbs, adjectives or modal verbs. Some of these qualifications are normative, and

[5] (Mally, 1926). In (Mally, 1971), p. 243.
[6] Cf. (von Wright, 1968), p. 12: "It is important to distinguish – more sharply than it is often done – between deontological or normative concepts on the one hand and axiological or value concepts on the other hand. Examples of value concepts are notions of good and bad(evil)…The important notion of preference is also axiological. "
[7] E.g. (Hansson, 1970)
[8] E.g. (Rescher, 1966).
[9] See Historical Points, below.

may be ruled by specific logical principles. The main theories in this area are variants and extensions of standard deontic logic.

Specific theoretical problems, often called *deontic paradoxes*, which are partly problems of formalisation, partly problems of formal semantics, have been tried solved by several of the research programmes mentioned above. One solution, has been to demand additional formal resources, such as special conditionals[10], quantification[11], preference relations, temporality[12], dynamic perspectives, agent perspectives,[13], two-sorted calculi to differentiate practical and theoretical language,[14], and so on. Focus on the fact that normative principles may be overridden or defeated by others has stimulated research in para-consistent and/ or non-monotonic logics for normative language, called defeasible deontic logic.[15]

In this first chapter of the thesis, I will study some of these logical theories from a historical point of view. I have written *a* brief and incomplete history of logics formalising normative language from Mally to defeasible deontic logic. It may be dry reading at times as logic often is. As a reward, towards the end of this long chapter the reader is offered an answer to the important question, whether it can be obligatory that every good kid gets a present from Santa Claus every year, and yet that there is not a single kid which is permitted to get one.

Principles of the Historical overview

The sources do not lend themselves to any obvious thematic taxonomy. The historical survey is thus mostly, but not strictly, chronological. The logics treated before von Wright may be characterized as non-modal, which is the reason (Rescher, 1966) comes before (von Wright, 1951) in the treatment. These logics try to model normative language with a formal language employing a finite number of logical values or even by just applying classical propositional logic to normatives. (The theorems of Mally´s logic turn out to be reducible to theorems of classical propositional logic). The logics treated after and including (Von Wright, 1951) may mostly be characterized as modal logics, but some recent defeasible logics are not. Also, Castañeda was strictly against having his deontic logics characterised as modal logics.[16] The treatment of SDL and quantified SDL is more thematic involving the contributions by several theorists, Prior, Castañeda, Chisholm, Hintikka, Forrester,

[10] E.g. Von Wright´s Dyadic Deontic Logic.
[11] E.g. (Hintikka,1971).
[12] E.g. (Thomason, 1981).
[13] E.g. (Horty, 2002).
[14] E.g. (Castañeda, 1981).
[15] E.g. (Nute (ed.), 1997).
[16] He also seems to have been against possible world semantics in general. See section on first-order deontic logics.

Sergot, Prakken and so on. Otherwise the method simply consists in going through theorists one by one. It is not the least boring approach, but since I knew of no thorough treatment of the history of modern normative logic to use for guidance, I wanted to err on the side of caution with regard to possible conceptual connections between the sources. The ones I have found myself I have noted. I consider even an incomplete attempt as this a step in the right direction. Sometimes the philosopher in me takes over and my interpretation becomes somewhat inventive and creative. I usually warn the reader by writing "critical commentary" before this happens. I will have to note though, that the final part of the treatment of first-order SDL cannot really be called historical. It is rather a creative elaboration of some of Hintikka´s ideas. To be more specific the scapegoat example and the Santa Claus example will not be found elsewhere in the literature. At the very end I have included a general evaluation of the sources treated, justified as a result of the (supposed) insight won by my labour.

I will sometimes work with the following *descriptive formalisation* in the following.
Uppercase letters starting from Q will be used to denote imperatives and deontic sentences.
Lowercase letters starting from q will be used to denote propositions.

A Few Historical Points About Normative Logic Before 1926

Aristotle discusses a practical syllogism, which he distinguishes from a theoretical syllogism.[17] The conclusion of a practical syllogism is an action. Stoic philosophers distinguished propositions (bearers of truth) from questions and commands.[18] The stoic philosopher Chryssipos wrote a treatise entitled "On Commands," and according to Diogenes Laertius´ this was a logical work.[19] According to Simo Knuuttila (Knuuttila, 1981) fourteenth century logicians were aware of deontic equivalences such as: When something is not permitted it is equivalent to it being obligatory that it is not done.[20] Leibniz paid attention to legal modalities. David Hume has been taken to claim that normative sentences cannot be derived from descriptive sentences. This principle is sometimes called Hume´s guillotine, because it separates normative and descriptive language.
These were just a few disjoint points from the early history of logics for normatives. Even these rudimentary remarks show that the area was not neglected in the history of western logic. I now turn

[17]. (Aristotle. *The Nicomachean Ethics*): 1140a 31, 1144a 31, 1147a 2. Cf. (Hare, 1952), p. 26, footnote.
[18] (Mates, 1953), p. 19.
[19] (Rescher, 1966), pp. 5-6. Diogenes Laertius. *Lives of Eminent Philosophers:* 7, 191.
[20] (Knuuttila, 1981). In (Hilpinen (ed.), 1981), p. 236.

to (Mally,1926), which is the first work of logic for normatives employing modern formal techniques.

Ernst Mally

Mally was the first to develop a formal logic for normatives. He called the result *Deontik*, derived from the Greek δεόντως, meaning "as it should be".[21] To a language of classical logic he introduced an operator ! . For a formula B expressing a state of affairs !B means that this state of affairs is obligatory. Another dyadic operator *f* (read in German *forder*s, "commits (you) to") is defined as (A→B!) and used to express conditional obligations (A *f* B), described by Mally:

> Wenn A (besteht, zutrifft), so soll B sein.[22]

His approach is axiomatic, as it was common in his day. The first axiom of his system (Grundsatz I) is the following:

$((A f B) \wedge (B \rightarrow C)) \rightarrow (A f C)$[23]

In the undefined form it amounts to

$((A \rightarrow !B) \wedge (B \rightarrow C)) \rightarrow (A \rightarrow !C)$.

Mally called it a *Grundsatz der Folgerichtigkeit* and meant it to ensure the following:

> Wenn ein Sachverhalt sein soll, so soll jedes Implikat dieses Sachverhaltes sein.[24]

The idea is to commit us to the consequences of our actions, and different variants of this principle have been discussed widely in the literature since. It is not clear what kind of implication Mally had in mind for (B→C), material or strict.[25] Since Mally, the principle is often thought to hold for strict implication, so that only what logically follows from a required action is required. Let **O**B mean that B is obligatory, and let ├ B mean that B can be derived in a logical system. The principle can then be written:

1) If ├ (B→C) then ├ **O**B→**O**C, called the *principle of inheritance of obligations*[26]. I return briefly to Mally´s *Grundsatz I* in the second part of this thesis. Mally presented four other principles or axioms. They were shown to imply B↔!B, that an obligation is equivalent to the fact it demands and it has been pointed out that this

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[21] (Føllesdal, Hilpinen, 1970), p. 1.
[22] (Mally, 1926). In (Mally, 1971), p. 243.
[23] (Mally, 1926). In (Mally, 1971), p. 246.
[24] (Mally, 1926). In (Mally, 1971), p. 246.
[25] (Føllesdal, Hilpinen, 1970), p. 6.
[26] (Nute, Yu, 1997). In (Nute (ed.), 1997), p.3.

…makes deontic logic trivial. [27]

As a consequence of Mally´s work quite a bit of research was conducted in the 1930s and 1940s in the field of normative logic. The next stop we make is by Jørgensen, who denied the possibility of a logic of imperatives.

Jørgen Jørgensen

The Danish philosopher Jørgen Jørgensen thought that inferences containing imperatives seemed possible. [28] He gave the following schemas as examples.

2)

Keep your promises!	Love your neighbour as yourself!
This is a promise of yours	Love Yourself!
___	___
Keep this promise!	Love your neighbour![29]

Yet, Jørgensen denied the possibility of an imperative logic. He denied that imperatives can be part of inferences because they lack truth-value. His argument is thus based on logical inference considered as logical consequence. The classical semantic account of logical consequence is that, if the premises are true, then the conclusion cannot fail to be true. Since imperatives cannot be true or false, they cannot be part of a logical inference.[30] He sums up his position in form of a puzzle:

> So we have the following puzzle: According to a generally accepted definition of logical inference only sentences which are capable of being true or false can function as premises or conclusions in an inference; nevertheless it seems evident that a conclusion in the imperative mood may be drawn from two premises one of which or both of which are in the imperative mood.[31]

The proposed solution to this puzzle (the puzzle is sometimes called *Jørgensen´s Dilemma),* is given by Jørgensen through a distinction between *an imperative factor* and *an indicative factor* of an imperative. The indicative factor is an indicative sentence derived from the imperative sentence. It is the sentence, which describes what is prescribed by the imperative.[32] Here is the basis for a logic of fulfilment of imperatives, discussed below.

> ...the derived indicative sentence can be dealt with(...) according to the rules of ordinary logic, which do not apply to the imperative sentences.[33]

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[27] (Føllesdal, Hilpinen, 1970), p. 5.
[28] Jørgensen, Jørgen. (1937). Imperatives and Logic. *Erkenntnis* 7: 288-296.
[29] Cf. (Jørgensen, 1937), p. 290.
[30] I use "inference" in a broad sense. Logical consequence is a semantic account of inference, logical deduction is a syntactical. Jørgensen´s argument is based on the semantic account, and in part II of this thesis I widen this account to include imperatives.
[31] (Jørgensen, 1937), p.290.
[32] (Jørgensen, 1937) , p.291.
[33] (Jørgensen, 1937), pp. 291-292.

A second point, argued by Jørgensen, was that imperatives only get their *meaning* through the indicative sentence “derived” from it.[34] Only the indicative sentence may be verified and only verifiable sentences have meaning. This account of the meaning of imperatives was in accordance with the accounts of other logical positivists such as Schlick and Menger.

Alf Ross

The Danish legal philosopher Alf Ross supplied many original logical insights in his article (Ross, 1941) .[35] The central question of his investigation was:

> Can an imperative be a constituent part of a logical inference?[36]

He used the term imperative as a broad term, including both imperatives and deontic sentences.[37] He did not accept the presupposition of Jørgensen´s argument.

> The concept of the logical element is not therefore, as assumed by Jørgen Jørgensen, necessarily attached to the logical values truth and falsity, and imperatives are not therefore precluded beforehand from being constituent parts of a logical inference.[38]

The question must be answered, whether there can be an analogous set of logical values for imperatives.[39] Ross discusses four different solutions to this question. The main contestants for logical values correspond to two features of imperatives, which will show up again and again. They may be *in force* (Ross says that they are *valid,* which is a regrettable term from a modern point of view, since it is used in such important other contexts), leading to values of being in force. They may be fulfilled or "carried out" (Ross uses another regrettable term and says they may be *satisfied)* leading to fulfilment values. Ross says that the feeling of evidence connected with inferences such as the ones in 2) above, are connected with the imperative being *in force*, not the imperative being *fulfilled*. [40]

> ...the satisfaction of the imperative is not the only and presumably not even the most "natural" analogy to the truth of the indicative.[41]

He first concentrates on the more "natural" solution of an imperative being in force. The first solution would be to ascribe objective imperative force values to imperatives, a solution Ross rejects, perhaps because he associates it with an objective morality.

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[34] (Jørgensen, 1937), p. 291.
[35] Ross, Alf. (1941). Imperatives and Logic. *Theoria*: 53-71.
[36] (Ross, 1941), p. 54.
[37] (Ross, 1941), p. 55.
[38] (Ross, 1941), p.59.
[39] (Ross, 1941), p. 59.
[40] (Ross, 1941), p.61.
[41] (Ross, 1941), p.61, footnote.

> The belief in an objective validity has its place in the lumber room of religious-moral metaphysics.[42]

The second solution is to ascribe fulfilment values to imperatives, the third is to ascribe subjective imperative force values, and the fourth is a combination of the two, which he rejects. I will take the solutions one by one.

His second solution is a logic of fulfilment, giving fulfilment values to imperatives. To fully appreciate this it must be mentioned that Ross agrees with Jørgensen´s basic semantic analysis of the imperative sentence into an indicative and an imperative element. The indicative element is primary since it contains the semantic content also called *the theme of command* by Ross. I will call this *the standard analysis of the imperative.* The standard analysis in turn gives rise to a syntactic representation that I call the *operator analysis of imperatives*. This looks like this:

I=!(S) where I is an imperative, and S is a proposition and ! is an *imperative operator*, which turns a proposition into an imperative. S is what from now on will be called the *corresponding proposition* to an imperative in this thesis. For instance, I may be "Send the letter, Andrea!". Then S is "Andrea sends the letter".[43] From the standard analysis of the imperative a logic of fulfilment of the imperative follows easily. The idea is that when S is true I is fulfilled. When "Andrea sends the letter" is true, "Andrea send the letter!" is carried out or satisfied or fulfilled. It is possible to disagree with the standard analysis of the imperative, and still agree that there is this relationship between an imperative and a corresponding proposition, that the imperative is fulfilled when the corresponding proposition is true.[44] Ross´ thoughts on an imperative being in force, seem to be of the same sort as the ones that lead to deontic logic. The operator ! (he sometimes uses the letter I) can occupy different places in a sentence. So he distinguishes $!(p\vee q)$ from $!p\vee !q$ for instance.

The third solution is to take an imperative being in force (Ross: valid) as an *expression of a psychological state*, either a *state of demand* (in the *issuer*) or a *state of acceptance*(in the *addressee).* The fourth solution is a combination of a logic of fulfilment and a logic of being in force, but he holds this to be impossible, though desirable.[45]

He now continues by comparing his logic of fulfilment with his logic of being in force. I will not go through all of this, but give some examples.

Negation. The operator analysis of imperatives gives rise to two types of negation, namely

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[42] (Ross, 1941), p. 60.

[43] From looking at examples like this, it is intuitively quite clear what is meant by an imperative having a corresponding proposition. Theoretically, it can be a quite problematic idea, though.

[44] This is what I do in part II of this thesis.

[45] Ross often confuses logical with psychological considerations, but I have widely ignored this fact.

3)$\neg$!p and 4) !$\neg$p. The first of these, Ross claims, corresponds to a logic of being in force, and it means that, the imperative !p is not in force, which means that you do *not* have to do p.

4) corresponds to a logic of fulfilment and it means that the imperative $\neg$p is in force, which means that you have to do *not* p, or abstain from doing p. He notes that imperatives in the grammatical sense can only express instances of 4), that is e.g. "do not close the door!".[46] He argues, that from !p being fulfilled we can conclude that !$\neg$p is not fulfilled, and vice versa. He argues that from !p being in force we can conclude that $\neg$!p is not in force.[47] He furthermore argues that the intended purpose in practical inference with regard to negation is what he calls a combination of the two logics. This combination would allow us to go from !p being in force to $\neg$!$\neg$p. If you have to close the door, then you do not have to not close the door. The problems of the two kinds of negation were clarified by von Wright´s Monadic Deontic Logic. To anticipate matters a bit, representing the full range of normatives (obligations, prohibitions, permissions) requires both kinds of negation and they are both related to the force of the normative. Going from !p to $\neg$!$\neg$p corresponds to the principle **O**P$\rightarrow$**P**P in von Wright´s logic. (see the square of opposition for deontic sentences, figure 1 below).[48]

Ross´ Disjunctive Paradox

Ross´ article is most famous for his disjunctive paradox.[49] In a logic of fulfilment it means that we can go from "Send the letter" is fulfilled ("The letter is send" is true) to "Send the letter or burn it" is fulfilled ("The letter is send or burned" is true). In a logic of being in force it means we can go from "Send the letter" being in force to "Send the letter" being in force or "burn it" being in force. The first he represents as going from !p to !(p$\vee$q) the second as from going from !p to !p$\vee$!q. The conclusion of the first is an imperative with a disjunctive theme of command the other is a disjunction of two imperatives with differing themes of command.

Ross´ Conditional Fallacy

At least as important as Ross´ disjunctive paradox are his considerations about conditionals, even though this is sometimes neglected when discussing Ross. He distinguished two kinds of normative conditionals. What I will call *Ross´ conditional fallacy* is the following "inference".

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[46] (Ross, 1941), p. 63.
[47] (Ross, 1941), p.64.
[48] (Segerberg, 1990) is an attempt at formalising some of Ross´ ideas directly using tools from modal logic.
[49] This is usually just called *Alf Ross´ paradox.* In light of *Ross´ conditional fallacy* below, I have added "disjunctive" to the name.

5) Love yourself!

If you love yourself, you are also to love your neighbour!

____*

Love your neighbour!

Ross states two things. One is that this inference is clearly valid in a logic based on fulfilment. If "Love yourself!" is fulfilled then "you love yourself" is true. This is the antecedent of the conditional second premise, giving us the consequent with modus ponens. But, as Ross says:

> ...naturally understood, the conclusion appears to be evidently false, because the intermediate concept is lacking.[50]

And further:

> For the second premise says that one is to love ones neighbour in the same way as one *actually* loves oneself. If therefore the first imperative is not satisfied, which ahs not been presupposed here, then the conclusion is false. [51]

Ross conditional fallacy presents a problem to any logic of fulfilment, such as Rescher´s logic of commands (see below). It must be distinguished from two other forms of conditional reasoning, which are valid.

6) You are to love yourself!

If you are to love yourself then you are to love your neighbour!

____

You are to love your neighbour.

7) You love yourself.

If you love yourself, then you are to love your neighbour!

____

You are to love your neighbour.

Using the upper-case letters to denote normatives lower-case letters to denote propositions we can distinguish the three forms of modus ponens 5), 6), and 7) as follows, where q is the corresponding proposition for Q. 5.1 is *Ross´ conditonal fallacy.*

5.1) $Q \rightarrow R$

q

________*

R

[50] (Ross, 1941), p. 66.
[51](Ross, 1941), p. 67.

5.1) must be distinguished from 6), (Q→R and Q implies Q) and also from 7), (q→R and q implies R). Ross´ conditional fallacy has sometimes been mentioned in the literature. In (Castañeda, 1960), for instance, he presents an instance of Ross´conditional fallacy similar to 5) above and claims it invalid for the following reason:

> …the possibility that the imperative be unfulfilled is included in the validity claim of the imperative inference…[52]

W.D. Ross

Alf Ross must not be confused with W.D. Ross who introduced the important distinction between actual and Prima Facie Obligations.[53] A Prima Facie obligation is an apparent obligation. If it is not overridden by other obligations it is turned into an actual obligation. This distinction is very important in defeasible deontic logic, as we will see below.

R.M. Hare´s The Language of Morals

> In order to characterize clearly the difference between imperatives and indicatives, it will be helpful to analyse the two types of sentences, as to make it plain what elements of meaning they have in common, and so isolate the essential difference.[54]

I will focus on aspects of Hare´s complex theory that I feel are the most important for my discussion about normative logic. R.M. Hare argues that imperatives cannot be reduced to propositions about the future. They cannot be reduced to expressions of wish or emotions, as Ross and the positivists would have it.

> Such an interpretation, when applied to such sentences as "Supply and fit to door mortise dead latch and plastic knob furniture", is unplausible.[55]

You can carry out a command written in a set of instructions without there being any authoritative person desiring for you to do that. When you get a letter from an official authority telling you to pay your taxes that order cannot be reduced to somebody desiring for you to pay the rest of your taxes, (The letter may be written by a computer, and no living person may even be aware, that you have not paid all your taxes). Normative language cannot be reduced to a psychological state of demand

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[52] (Castañeda, 1960), p. 28.
[53] Ross, W.D.. (1930). *The Right and the Good.*
[54] Hare, R.M.. (1952). *The Language of Morals:* 17.

in a norm giver or imperator. On the other hand, you may be an addressee of an obligation that you do not endorse or even know about (e.g. as a stranger in a foreign country, where you do not know the laws). So, the normative language cannot be reduced to a state of acceptance or endorsement in an addressee either. In general, normative language cannot be reduced to psychological states.

Indicatives and imperatives have something in common. According to Hare
8) Shut the door.
9)You are going to shut the door.
Could be recast into the two sentences.
8.1) Your shutting the door in the immediate future, please.
9.1) Your shutting the door in the immediate future, yes.
The common element
10) Your shutting the door in the immediate future
he calls the *phrastic*, from Greek to point or to indicate. What is different in 8.1) and 9.1) he calls the *neustic* element, from Greek, to nod assent.[56]

> (1) The speaker points out or indicates what he is going to state to be the case or command to be made the case;(2) He nods, as if to say "It is the case" or "Do it". He will, however, have to nod in a different way, according as he means one or other of these things.[57]

He stresses that the common element is no more "propositional" than it is "imperative".
He says about the logical connectives:

> they are common ground between indicatives and imperatives.[58]

The same holds for the quantificational words "all" and "some". I regard this semantic analysis a step forward from Alf Ross´ and Jørgensen´s propositional theme of command with an added imperative operator. A syntactic parallel would need two operators a propositional and an imperative, added to a common semantic content. But if logic is applied to the common content only, Hare notes, how come we cannot just change the operators around at will?
Hare also claims that imperatives can be contradictories and gives as an example "Shut the door" and "do not shut the door!"[59]
As a consequence of this, Hare thinks that imperative logic is bivalent, in the same way as classical propositional logic is. Here is one of his reasons.

> It is quite clear that if I do not say "Shut the door!" this does not compel me, logically, to say"Do not shut the door". I can say "You may either shut the door or not shut the door"; or I can say nothing at all. But similarly, if I do not say "You are going to shut the door" this does not compel me logically to say

55(Hare, 1952), p. 10.
56(Hare, 1952), pp. 17-18.
57(Hare, 1952), p. 18
58(Hare, 1952), p. 21.
59(Hare, 1952), p. 23.

> “You are not going to shut the door”. I can say “You may be going to shut the door, and you may be going not to shut the door” or I can say nothing at all. (...) The truth is that our language possesses ways of speaking in a three-valued way and ways of speaking in a two-valued way(...). ...simple imperatives are normally two-valued.[60]

He gives as an argument that the following imperative is analytic:

> At your next move, either move your queen or don´t move your queen.[61]

Which imperative, he says, gives no positive instructions, just as the proposition: “It is either raining or not raining” says nothing about the weather.[62]
and concludes,

> If the logic of simple imperatives were three-valued, the sentence quoted would not be analytic.[63]

Hypothetical imperatives

By Alf Ross we find the following statement about,

> ...the so-called hypothetical imperatives, e.g., “If you desire to make water boil, you must heat it to 100°”. This sentence is identical as to meaning with the descriptive proposition that water does not boil until heated to a temperature of 100°.[64]

As an example of a hypothetical imperative[65] Hare has,
11) If you want to go to the largest grocer in Oxford, go to Grimly Hughes.
Hare claims this is logically equivalent to
12) Grimly Hughes is the largest grocer in Oxford.[66]

Critical comment. It would be a job for a first-order logic with equality and singular terms to show whether 11) and 12) are really equivalent.[67] My logical intuition is that whereas 12) entails 11), 11) does not entail 12). Grimly Hughes could denote something else than the largest grocer. Consider the hypothetical imperative:
13) If you want to go to the town hall, go to the town square.

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[60](Hare, 1952), pp. 23-24.
[61](Hare, 1952), p. 24.
[62](Hare, 1952), p. 24.
[63](Hare, 1952), p. 24.
[64](Ross,1941) , p. 54.
[65] This is not the same as Immanuel Kant´s distinction between hypothetical imperatives and categorical imperatives, between demanding “with a purpose” and “objectively”, which is an ethical and not a linguistic distinction. (Kant, 1785), p. 34: Alle *imperativen* nun gebieten entweder *hypothetisch* oder *kategorisch* .Jene stellen die praktische Notwendigkeit einer möglichen Handlung als Mittel zu etwas anderem, was man will (oder doch möglich ist, dass man es wolle), zu gelangen vor. Der kategorische Imperativ würde der sein, welcher eine Handlung als für sich selbst, ohne Beziehung auf einen anderem Zweck, als objektiv-notwendig vorstelle.”
[66](Hare, 1952), p. 34.
[67] I will not present such a strong logic in this paper.

We do not want to infer that:

14) The town hall is the town square.

A first-order logic might show this. It might also show that since 11) does seem to follow from 12) Hare´s principle,

> *the rule that an imperative cannot appear in the conclusion of a valid inference, unless there is at least one imperative in the premises...*[68]

does not hold. To his defence, he did exclude the hypothetical imperatives from the principle, but this seems rather arbitrary.[69] Another point is that Hare thought that the word "want" in hypothetical imperatives stands for

> …an imperative inside a subordinate clause.[70]

This view is problematic, since it reduces intentions to self-orders, and these are clearly different entities.[71] The distinction is similar to the one between making a statement and believing one. Time has now come for looking at a logic of fulfilment in more detail. The most ambitious attempt at carrying out this project that I know of, is (Rescher, 1966), which I turn to next.

Nicholas Rescher´s Logic of Commands

In *The Logic of Commands* Nicholas Rescher proposes a logic for certain imperatives. The term command is taken to cover:

> ...orders, directives, injunctions, and prohibitions...[72]

But not requests or advice, which have different functions in language from commands.[73] The term command is furthermore not to be understood as the specific giving of a command but as the

> …abstract meaning-content that is conveyed when this command is given…[74]

Rescher also calls this an *instruction* conveyed by a command. [75]This is thought in exact analogy to propositional logic, where the proposition is the meaning-content of any specific assertion, which

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[68](Hare, 1952), p. 32.

[69]In the case of deontic logic, A.N. Prior showed that $Op \vee q$ is implied by q, and that Op is implied by $q \wedge \neg q$. These inferences hold in the logic of mandates presented in part II of this thesis as well.

[70](Hare, 1952), p. 32

[71] Cf. (Castañeda, 1975), p. 43: …we are *not* here reducing intentions to prescriptions, much less to mandates or imperatives…

[72]Rescher, Nicholas. (1966). *The Logic of Commands*: 2.

[73](Rescher, 1966), p.1.

[74](Rescher, 1966), p. 9.

[75](Rescher, 1966), p. 9.

expresses this content. Logical relations are confined to hold between commands understood this way, not between specific utterances of commands.[76]

Rescher concludes three things about the logic of commands:
15) It must possess temporal resources.
16) The concept of validity of a command inference must be understood as a question whether a conclusion is implicitly contained in a set of instructions.
17) The semantic theory yielding validity and other logical concepts must be made via the bridging link of a command termination statement. (the corresponding proposition of an imperative).[77]
The key question is the question of validity, and Rescher´s goal is to find an adequate theory for imperative validity. An informal analysis of commands is conducted as a preliminary to obtaining this goal. He first distinguishes three elements of the command, *the source* (who issues the command), *the recipient*(or addressee, to whom the command is given) and *the mooted action or result,* which is described as central to the command:

> The pivotal component of a command is its mooted action or result - the possible process of activity or state of affairs which the source enjoins the target to do or achieve or to refrain from doing or achieving.[78]

To this is added *execution timing* (do it now and do it always or standing orders), *execution-precondition* and *period the command is in force.*[79] Except the last one, the period a command is in force, all these elements are carried through to the syntactical representation of commands.
Rescher´s notation is as follows:[80]
Temporal variables: t, t´, ... Command addressees: X,Y,Z,X_1.... Command requirements or Mooted actions: A,B,C,A_1.... Command Execution preconditions: P,Q,R,P_1,...

Syntactically, Rescher introduces a specific kind of conditional as well formed and his project is thus not far from dyadic deontic logic, which he indeed helped develop in the early 1960s.[81] The

[76](Rescher, 1966), p. 8.
[77](Rescher, 1966), pp. 123-124.
[78](Rescher, 1966), p. 18. The distinction between a state which must be realized(e.g. "Learn French!") and an action which must be performed(e.g. "Raise your hands!", which Rescher makes here is not carried through in the rest of the book in a way relevant to my discussion.
[79](Rescher, 1966), p. 21, p. 23, p. 25.
[80]I use the same set of standard logical signs as elsewhere in this thesis.
[81] Cf (Lemmon, 1965), about (von Wright, 1956) and dyadic deontic logic p. 42: The system was enriched by Rescher, attacked by Anderson, modified by Rescher…

main difference is that the logic of commands is not modal, but has four or five (see below) semantic values. A modal semantics, on the other hand, cannot do with a finite set of values without relating values to points in a frame as in Kripke semantics.[82] There are two basic formulas with different execution timings

X!A/(P) means, that *the next time* when condition P is true the order A is in force for X.

X!A/P means that *anytime* when condition P is true the order A is in force for X.

As an example take the order "John, next time you see Paul tell him I am coming to the party". The mooted action which we symbolise B is "tell him I am coming to the party". The addressee John is symbolised Y and the pre-condition "You see Paul" is symbolised Q. The whole order is then symbolised Y!B/(Q).

The logical connectives can be applied to addressees, command requirements and preconditions. As a special case the order X!A/* just means that A(categorically, atomically) is in force for A. * is thought of as an empty clause or tautology such as the sun is shining or not.[83] The line in the order -!A/P represents the second person pronoun "you", as in "you, open the door!".

The next step is to find a semantic theory that fits the syntax. Rescher accomplishes this via the idea of command termination.

If a command C= X! A/P is given at some time t, one can form the corresponding (purely assertoric command termination statement: "Tt(C)" for "X has always realized A whenever P obtained after time t". [84]

As an example for the command "Tom, tell Jim to see me when (next) you see him" the corresponding command termination statement is "Tom did tell Jim to see me when next he saw him". The step from command to command termination statement is not always straightforward, and standing orders are only terminated when the addressee is dead.[85]

Termination is distinguished from obedience as a behavioural rather than psychological predicate. The logical theory is based on the concept of command termination.[86] This is done in the following way. Rescher defines two orders C and C1 to be T-inconsistent (termination inconsistent) if it is not possible to have C∧C1. [87] We could now expect entailment to be defined from this as C1 entails C

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[83](Rescher, 1966), p. 39.
[84](Rescher, 1966), p. 52.
[85](Rescher, 1966), p.53
[86](Rescher, 1966), p. 57.
[87](Rescher, 1966), p.59.

if it is not possible that C and not C1. This is indeed the case, but Rescher adds that a command requirement not only can be terminated or the opposite, it can also simply be bypassed. This happens when the precondition never arises. In the command above for instance, if Tom never sees Jim, then the command requirement to "Tell Jim to see me" is bypassed. Rescher finally decides to adopt the definition of validity that whenever the premises are terminated, then the conclusion is terminated or bypassed. The five semantic values he can be seen to operate with are terminated, violated, bypassed (for command requirements) and true and false (for command preconditions). Rescher bases his logical inferences on a concept of command decomposition and command coverage. A command X!A/P *covers* another command Y!B/R, when Y is a subset of X if they are groups or X=Y when they are individuals, when the action B is included in A(basically $A \rightarrow B$ would be a theorem of classical logic if A and B were propositions), and when $P \rightarrow R$ is a theorem of classical propositional logic. Commands can be *decomposed* in three ways corresponding to the syntactic components X, A, and P. We can see these as basic logical and set-theoretical relations and represent them

$Y \cup X!A \wedge B/p \vee q$ may be decomposed into $X!A \wedge B/p \vee q$, $Y!A \wedge B/p \vee q$ (addressees decomposed) $Y \cup X!A/p \vee q$ $Y \cup X!B/p \vee q$ (command requirement decomposed), $Y \cup X!A \wedge B/p$, $Y \cup X!A \wedge B/q$ (command preconditions decomposed). The last two decompositions work because $p \rightarrow (p \vee q)$ is a theorem of classical propositional logic and similarly for q.[88] Decompositions may also be combined. Validity is now defined through command coverage and decomposition. First *patent invalidity* of an inference is defined as:

> It is possible for all of its assertoric premises to be true and for all of its command premises to be terminated, and yet its command conclusion to remain unterminated.[89]

Patent validity is defined

> The inference whose conclusion is the command C and whose premises include the command C1,...Cn, is patently valid if the command C can be decomposed into the set of commands C1#,...Cm# in such a way that each Cj# is covered by some of the Cj.[90]

As an example of patent validity he gives the following.

18) C1= X!A&B/*

C2=X!C&D/P

____

C3= X!A&C/P

[88](Rescher, 1966), pp. 68-69.
[89](Rescher, 1966), p. 78.

This can be seen in the following way. C3 can be decomposed into C4= X!A/P

C5= X!C/P(commands decomposed). C4 is covered by C1 because X=X, A$\wedge$B includes A, and (P$\rightarrow$*) is a tautology because * is. (Thus, if something is to be done under tautologous circumstances it is to be done under *any* (specific) circumstance.) All the requirements of command coverage is fulfilled. C5 is covered by C2 in the same way. Hence the inference meets the requirements for being patently valid.

Interpreting this schema with some sentences we get:

18.1) John, always be friendly and smile!
John, always say hello and clap your hands, when you see the president!
____
John, Always be friendly and clap your hands when you see the president!

Another schema that comes out valid according to this definition of validity is:

19) X!A/(You do B)
X!B/(*)
____
X!A/(*)

19) is *Ross´ conditional fallacy*(see above). It is a problem for Rescher´s theory and a challenge for any logic where validity is based on the fulfilment of imperatives that this schema comes out valid. We have now had a closer look at a logic of fulfilment for imperatives. It is time to turn to deontic logics, logics created to represent all the normative categories (permissions, prohibitions and obligations).

Georg Henrik von Wright´s Monadic Deontic Logic

Georg Henrik von Wright´s paper *Deontic Logic*(1951) initiated the phase of using techniques from modal logic to model normative language.[91] He writes

> ...the system of deontic logic(...)studies propositions(...) about the obligatory, permitted, forbidden,..[92]

A problem of special interest to von Wright was the so-called decision problem:

> A task of particular importance which deontic logic sets itself is to develop a technique for deciding, whether the propositions it studies are logically true or not.[93]

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[90](Rescher, 1966), p. 79.
[91]Von Wright, Georg Henrik. (1951). Deontic Logic. *Mind*, vol.60:1-15. Reprinted in (Von Wright, 1957): 58-74.
[92](Von Wright, 1957), p. 62.
[93](Von Wright, 1957), p. 62.

In fact, von Wright´s only concern in this paper is deontic propositions which are true on *formal grounds*.[94] I call the system monadic deontic logic or MDL.

To solve the decision problem on formal grounds for propositions about what is obligatory, permitted and forbidden, von Wright developed his deontic logic in parallel to alethic modal logic. The possible, necessary and contingent are called alethic modalities (modes of truth).The forbidden, obligatory and permitted are called deontic modalities.[95] He introduces operators for the deontic modalites **P**(permitted), **O**(obligatory), **F**(forbidden). **P** is taken as primitive and the others are defined.(See 21) and 23) below).

There are two levels in von Wright´s deontic logic. The deontic sentences are the second level, and they are considered propositions with a truth-value. The first level is the level of *acts*. They are the answer to the question:

> What are the "things" which are pronounced obligatory, permitted, etc?[96]

When a deontic operator is used on an act a deontic proposition is created. The atomic variables, P, Q,… do not denote propositions or truth values in MDL but classes of acts.[97] Von Wright distinguishes between act-properties and individual cases which fall under the properties. As an example "stealing" is an act-property, and "Tom steals a car" is an individual case of stealing. Individual cases are also called act-individuals.[98] Von Wright decides to limit himself to act-properties, and the atomic variables represent these.[99] The formulas of von Wright´s system can be read as impersonal, applying to anybody . But Geach thinks that:

> …a more fruitful reading of which the symbolism equally allows would take *some one* agent´s obligations to be our subject matter.[100]

Sometimes von Wright adds "(for an agent)" to his interpretations, leading in the direction Geach suggests. It can now be seen how using a deontic operator on an act-property can create a deontic sentence. Letting Q be a name for the act-property of stealing, **P**Q would be "Stealing is permitted"(for an agent). Atomic Act properties may be connected by sentential connectives to form

[94](Von Wright, 1957), p. 60.
[95](Von Wright, 1957), p. 58.
[96](Von Wright, 1957), p. 58.
[97]I follow tradition and deviate from von Wrights´ original notation of using capital letters starting from A as atomic variables.
[98](Von Wright, 1957), p. 59.
[99](Von Wright, 1957), p. 59.
[100] (Geach, 1991) in (Geach (ed.), 1991), p. 34.

compound acts. The next step is to give the act-properties atomic or otherwise so-called *performance values*. He writes:

> ...The performance or non-performance of a certain act(by an agent) we shall call performance-values (for that agent).[101]

Performance functions are supposed to work between act-names in strict analogy to truth-functions for propositions.[102] The negation of the performing of an act is the act of not performing that act, and so on.[103] A tautology-act is an act that is always performed, which always has positive performance value, for instance the compound act of stealing or not stealing. A contradiction-act is one that is never performed, which never has positive performance value, for instance the compound act of stealing and not stealing.

Von Wright is naturally mostly interested in sentences, which are true in modal logic but not in propositional logic. That deontic logic cannot be reduced to the performance functions, that it is not performance-functional, he shows in the following way:

> From the fact that A is performed, we can conclude that not-A is not performed. But from the fact that A is permitted, we can conclude nothing as to the permitted or forbidden character of not A.(...) In the smoking compartment, e.g., not-smoking is permitted and also smoking. But in the non-smoking department not-smoking is permitted and smoking forbidden.[104]

In analogy to what is contingent which is defined in alethic modal logic as propositions Q and ¬Q both being possible ($\Diamond Q \wedge \Diamond \neg Q$)[105], what is morally indifferent is defined by von Wright as the acts Q and not Q both being permitted ($\mathbf{P}Q \wedge \mathbf{P}\neg Q$).[106] We saw that Ross had trouble with the types of negations of normatives. Von Wright´s analysis did a lot to clarify this, by showing that negation can be within the scope of the modal operator or vice versa, creating two types of sentences: A negated obligation or an obligation of a negated action, $\neg\mathbf{O}P$ and $\mathbf{O}\neg P$ and similarly for prohibitions and permissions. We might represent this and a few other logical relations with the *modal negation equivalences* that hold in von Wright´s logic and by a *square of opposition* for deontic sentences.[107]

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[101](Von Wright, 1957), p. 59.

[102] This strict analogy is questioned by (Føllesdal, Hilpinen, 1971) in (Hilpinen (ed.), 1971) p. 14: "…If an act of type *p* is performed and an act of type *q* is performed it does not follow that an act of type *p & q* is performed."

[103](Von Wright, 1957), p. 59.

[104](Von Wright, 1957), p. 63.

[105] Girle, Rod. (2000). Modal Logics and Philosophy: 9.

[106](Von Wright, 1957), p. 61.

[107] The square of opposition dates back to Aristotle, who made squares for quantified and alethic modal propositions.

Modal Negation Equivalences for Deontic Sentences

Some of the most important deontic negation equivalences are the following:

20) $\mathbf{P}Q \leftrightarrow \neg\mathbf{O}\neg Q$

21) $\mathbf{O}Q \leftrightarrow \neg\mathbf{P}\neg Q$ (Definition of **O** in MDL)

20) says that when something is permitted it is equivalent to its negation not being obligatory, and 21) that when something is obligatory it is equivalent to its negation not being permitted. For the forbidden we also have

22) $\mathbf{F}Q \leftrightarrow \mathbf{O}\neg Q$

23) $\mathbf{F}Q \leftrightarrow \neg\mathbf{P}Q$ (Definition of **F** in MDL)

Square of Opposition for Deontic Sentences

$\mathbf{O}Q$ $\neg\mathbf{P}\neg Q$ $\mathbf{F}\neg Q$	$\mathbf{O}\neg Q$ $\neg\mathbf{P}Q$ $\mathbf{F}Q$
$\mathbf{P}Q$ $\neg\mathbf{O}\neg Q$ $\neg\mathbf{F}Q$	$\mathbf{P}\neg Q$ $\neg\mathbf{O}Q$ $\neg\mathbf{F}\neg p$

Figure 1. Square of opposition for deontic sentences

Or, in words

Q is obligatory not Q is not permitted not Q is forbidden	not Q is obligatory Q is not permitted Q is forbidden
Q is permitted not Q is not obligatory Q is not forbidden	not Q is permitted Q is not obligatory not Q is not forbidden

In the square, the schemata within a box are logically equivalent, which can be verified by the modal negation equivalences above. The schemata in the top row are *contraries*, which means that they cannot both be true. It cannot be true that you have to close the door, and that you have to not close the door. On the other hand they may both be false. You may neither have to close the door nor not have to close it, in a normative sense. The schemata in the bottom row cannot both be false, they are *sub-contraries.* They might both be true, as in the smoking compartment example above. The schemata in the columns are *sub-alternatives*, which means that the top implies the bottom. The schemata across the diagonals are *contradictories,* which means that they have opposing truth-values. Von Wright´s logic obeys the logical relationships displayed in this square of opposition. For proofs the reader is referred to the section on standard deontic logic below. [108]

Most of the problems succeeding the appearance of MDL arose in connection with conditional obligations, or as von Wright calls it *commitment*. Commitment is described as follows by von Wright:

> Doing one act commits us to do another act, if the implication of the two acts is obligatory. For instance: giving a promise commits us to keep it.[109]

What is said here is represented by von Wright as **O**(P→ Q).

I will not present von Wright´s own method to show the problems that this notion of commitment brought but again refer to the following section on standard deontic logic.

Finally, it should be noted that von Wright´s logic is decidable, i.e. any formula will get a truth value through a specific decision procedure. [110] It should be clear what an achievement von Wright´s logic presented from what I have written, even though I have not presented this specific decision procedure. Instead, I will turn to what is called standard deontic logic, but the problems of standard deontic logic carry over to von Wright´s system, and they were indeed initially presented for von Wright´s MDL.

Standard Deontic Logic

Before turning to standard deontic logic, which is a branch of mainstream modal logic, I will present some general remarks about the history of modern, modal logic. Following (Blackburn, de Rijke, Venema, 2001), the history of modal logic can be divided into three phases.[111] A syntactic era (1918-1959), a classical era (1959-1972), and a modern era (1972 until present). C.I. Lewis´ work is taken as the starting point of formal modal logic.[112] The main work in this era consisted of exploring the consequences of axioms, and of proving their distinctness. Føllesdal and Hilpinen describe this type of work and the problems related to it for deontic logic.

[108] Cf. Fitting, Melvin and Mendelsohn, Richard L.. (1998). *First-Order Modal Logic*: 7-8.
[109](Von Wright, 1957)*,* p. 61.
[110] See (Hilpinen (ed.), 1971), pp. 10-11, for the specific details of von Wrights decision procedure.
[111] Blackburn, Patrick, and de Rijke, Maarten, and Venema, Yde. (2001). *Modal Logic:* 37-48.
[112] The authors note that Hugh McColl was before Lewis adding modal operators to propositional logic.

…a sample of consequences (or theorems) is derived from these axioms, and the adequacy of the axiom system is decided by considering these consequences. The plausibility of putative theorems is judged on the basis of the intuitive plausibility of their ordinary-language counterparts. [113]

However, intuitions are fickle and ordinary language is ambiguous. The classical era sees the development of *relational semantics*, as it is called by (Blackburn, de Rijke,Venema, 2001). [114] Stig Kanger (1957), Jaakko Hintikka, and Saul Kripke each began this work independently in the late 1950s and early 1960s. The idea is, in the modern language of (Blackburn, de Rijke, Venema,2001) to view modal logic as languages for talking about *relational structures*, mathematical entities consisting of a set together with a collection of relations defined on this set.[115] One major motivation is that with a formal semantics one gets a clear picture of what the formal language is intended to be about. Modal languages gives an internal perspective on the relational structures, because they speak about what can "be seen" from a point within a structure. The relation between relational semantics and proof systems was the main work of the classical era. Completeness and soundness theorems were the most important results. The discovery of incomplete modal logics gave rise to the modern era, a highly complex time where modal logic is used by computer scientists, philosophers and mathematicians alike to study relational structures, proof procedures and their ordinary language counterparts.

Standard deontic logic was created by a change in the interpretation of von Wright´s MDL. Instead of letting the atomic variables stand for generic names of actions with a performance value, they are taken to be propositions with a truth value. Sometimes, these are regarded descriptions of norms. This new interpretation has consequences for the syntax. Now atomic variables and compound sentences without deontic operators are well formed. So are sentences containing iterated operators.

This new interpretation has been criticised by some, who argue that the action perspective is fundamental for a clear understanding of deontic sentences.

…the topic is not what an agent is obliged to do or suffer or permit or abstain from but which states of affairs *ought to be.* Here, to my mind, the fatal step was taken. (…)…it was a false step.[116]

A similar criticism to Geach´s is found in (Horty, 2001). Perhaps nobody is as adamant about the necessity of this distinction between acts and states of affairs as Hector-Neri Castañeda. Consider also Prakken´s and Sergot´s *fence paradox* below, though.

[113] (Hilpinen (ed.), 1971), p. 15.
[114] Other names are Kripke semantics and possible world semantics.
[115] (Blackburn, de Rijke, Venema, 2001), p. xi.
[116] (Geach, 1990), p. 35.

The Relational Semantics of Standard Deontic Logic

Given what I have said above there is some anachronism in starting out by giving the formal semantics for standard deontic logic (SDL), as it was conceived in the syntactic era. The main idea of SDL was presented already in the early 1950s, although the name standard deontic logic is as recent as 1970.[117] Nonetheless, it is much easier to discuss SDL if I start out by simply presenting a formal semantics and a proof system. Jaakko Hintikka and Stig Kanger presented semantic accounts early for deontic logic. The version of the semantics presented below is in "Kripke-style" but the proof system in the next section is indeed "Hintikka-style" as it is analytic tableau. Basically, standard deontic logic draws deontic logic into the main stream of modal logic, since deontic operators operate on propositions with a truth value, and again the deontic sentences themselves are considered propositions with a truth value (like in MDL, but unlike imperatives, which are without a truth value). The truth value is not absolute, like in classical propositional logic, but relative to a point in a model, usually called a world, a state or simply a point. The basic idea is that an obligation is true at a point iff it is true at all points "deontically " accessible from that point. A permission is true at a point iff it is true at least one point accessible from that. The point where obligations and permissions are considered true are often identified with the actual world. The accessible points are called deontic alternatives, ideal worlds, or perfect worlds, because the propositions obligations operate on are all true in them. If **O**P is true at a world, P is true at all worlds accessible from that world. It is not strange, if what I just wrote is hard to comprehend. Apart from the oddity of accepting "ideal worlds" the main reason for this, in my view, is that the main semantic concept from normative language is not truth. We are interested in obligations and prohibitions being *in force* and being *fulfilled* or *violated*, and possibly in them being *overridden* or not. We do not have any clear intuition about what it is for an obligation to be "true". A remaking of the semantics in these terms would yield something like "an obligation is in force iff it is fulfilled in all perfect worlds". We cannot even begin to discuss how appropriate definitions like these are without speaking this way. The whole idea of a perfect world makes a lot more sense, if we can say: A perfect world is a point at which all the obligations in force in the actual world are fulfilled. Now, this reinterpretation is not possible for SDL, as we shall se below,(and it is instructive to see why not) but the reader should keep it in mind, especially since it is provides some of the motivation for the second part of this thesis. The point is that although relational semantics is very "liberal" and

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[117] Hansson, Bengt. An Analysis of Some Deontic Logics. *Nous,*vol.4, 1970: 373-398. Reprinted in (Hilpinen (ed.), 1971).

can show many different logical relations it has been very conservative in employing one specific semantic concept, truth. I think it is time to experiment with alternatives and develop formal semantics for normatives taking other semantic concepts as basic. It would probably require a philosophical theory of meaning to justify these concepts.

I will now present the standard formal semantics for SDL, following (Fitting, Mendelsohn, 1998).[118] The three main concepts are frames, models, and truth in a model. From these other semantic concepts, such as validity and consequence can be defined.

Formal Language

This is the language of standard deontic logic. It consists of a set of propositional letters, P,Q,R,… to symbolise propositions and five sentential connectives. One unary connective $\neg$ (not). Four binary connectives $\wedge$(and),$\vee$(or), $\rightarrow$(if…then…), $\leftrightarrow$(if and only if, iff) , and a set of parentheses. The formulas is the smallest set obeying the following conditions.

24.1) The propositional letters are formulas.

24.2)If X is a formula so is $\neg$X.

24.3) If X and Y are formulas and *b* is a binary connective, (X*b*Y) are formulas.

24.4) If X is a formula **P**X and **O**X are formulas.

Definition of *Frame*

A frame is the set of objects with a relation on them. We call the objects points or worlds and the relation the accessibility relation. Formally a frame consists of a non-empty set *g* and a binary relation $\mathfrak{R}$ on *g*. To have a single object to deal with we say that a frame is an ordered pair $\langle g, \mathfrak{R}\rangle$.[119]

Definition of *Model*

A frame is turned into a model through hanging formulas of the language on the points of the frame, relating formulas to points. We do this by adding another relation between formulas and the points of the frame. A model is based on a frame. Formally a model is an ordered triple $\langle g, \mathfrak{R}, \Vdash \rangle$, where $\Vdash$ is the relation between points and the formulas of the language. If Γ is a point and X a formula, $\Gamma \Vdash$ X, can be read: "X is true at point Γ" or "X is true in world Γ." When $\Gamma \Vdash$ p does not hold, we write *not*($\Gamma \Vdash$ p).

[118] (Fitting, Mendelsohn, 1998), pp. 1- 46.

[119] Cf. (Fitting, Mendelsohn, 1998), p. 12.

Definition of Truth in a Model

Let the relation $\Vdash$ be defined for the set of atomic formulas and the points of a frame.

A model is extended to evaluate all formulas of the language by the following recursive definitions.

25.1) $\Gamma \Vdash \neg X$ iff *not*($\Gamma \Vdash X$).

25.2) $\Gamma \Vdash (X \wedge Y)$ iff $\Gamma \Vdash X$ and $\Gamma \Vdash Y$.

25.3) $\Gamma \Vdash (X \vee Y)$ iff $\Gamma \Vdash X$ or $\Gamma \Vdash Y$.

25.4) $\Gamma \Vdash (X \rightarrow Y)$ iff if $\Gamma \Vdash X$ then $\Gamma \Vdash Y$.

25.5) $\Gamma \Vdash (X \leftrightarrow Y)$ iff $\Gamma \Vdash (X \rightarrow Y)$ and $\Gamma \Vdash (Y \rightarrow X)$.

25.6) $\Gamma \Vdash \mathbf{O}X$ iff for every $\Omega \in g$, if $\Gamma \mathfrak{R} \Omega$, then $\Omega \Vdash X$.

25.7) $\Gamma \Vdash \mathbf{P}X$ iff $\Vdash$ for some $\Omega \in g$, $\Gamma \mathfrak{R} \Omega$ and $\Omega \Vdash X$.

In SDL we put a restriction on the relation $\mathfrak{R}$. We want the relation $\mathfrak{R}$ to be *serial,* or in other words, to restrict ourselves to serial frames. The relation is serial iff every point has access to another point. This restriction means that SDL will obey all the logical relations displayed in the square of opposition for MDL above. It must be contrasted with the restriction to *reflexive* frames, (for any point Γ in a frame $\Gamma \mathfrak{R} \Gamma$) which would mean that any obligation is fulfilled in the actual world, and which is too strong a condition. This is the formal restriction on $\mathfrak{R}$:

25.8) For any point Γ in a frame there is a point Ω such that $\Gamma \mathfrak{R} \Omega$.

In the standard presentation of modal logic, this means that we go from the basic system K to the system D(for Deontic).[120] A further restriction to transitive, serial frames will take us to the system SDL4 or D4.

Validity Defined

The definitions above enable us to define validity of formulas. A formula is valid *in a model*, if it is true at all points in that model. A formula is valid *in a frame* if it valid in every model based on that frame. A formula is *a valid formula of SDL* if it is valid in every model of every serial frame.

A formula is thus not a valid formula if it is false at some point in some model based on some serial frame. The presentation of such a model is called a *counter example* to a formula (being valid).

If a formula is false at all points of all models of all frames it is *contradictory.*

A formula X is a *local consequence* of a set of formulas S if for any serial frame and for any model based on that frame and for any point Γ where all members of S are true, $\Gamma \Vdash X$.

X is a global consequence of a set of formulas U if for any serial frame and for any model where all members of U are valid(true at every point) we have $\Gamma \Vdash X$ for any Γ.
A set of formulas is satisfiable if they are not jointly contradictory.

I will now prove the close relation between the formula **O**P→**P**P and the collection of serial frames. I will prove that a frame is serial if and only if the formula **O**P→**P**P is valid in that frame. What this means, is that the collection of serial frames are precisely the ones where the formula **O**P→**P**P is valid.
I start out by proving that **O**P→**P**P is valid in every serial frame. According to the rule 25.4) above, this means that if the antecedent is true then the consequent is. Let Γ be any point in any model based on any serial frame.
To prove: $\Gamma \Vdash \mathbf{O}P \rightarrow \mathbf{P}P$.
Assume $\Gamma \Vdash \mathbf{O}P$ and that there is an Ω, $\Gamma\mathfrak{R}\Omega$. To prove: $\Gamma \Vdash \mathbf{P}P$.
Since we have $\Gamma \Vdash \mathbf{O}P$ and $\Gamma\mathfrak{R}\Omega$ we have $\Omega \Vdash P$. Because if we have **O**P at a point, then for every point accessible from Γ, we have P, and Ω is accessible from Γ. And since we now have established that we have $\Omega \Vdash P$, and we know that $\Gamma\mathfrak{R}\Omega$, it follows that $\Gamma \Vdash \mathbf{P}P$. Because according to the evaluation rule for **P**P above, if P is true at a point accessible from another point, then **P**P is true at the latter point, and this is exactly the case.

Now for the converse. To prove is that If **O**P→**P**P is valid in a frame $\langle g, \mathfrak{R}\rangle$, then it must be serial. Assume that **O**P→**P**P is valid in $\langle g, \mathfrak{R}\rangle$, but that $\langle g, \mathfrak{R}\rangle$ is not serial. I infer a contradiction. If $\langle g, \mathfrak{R}\rangle$ is not serial, then there is a point in a model based on that frame with no access to any other point. (Because the relation is serial exactly when there *always* is such a point). Since **O**P→**P**P is valid it follows from the definition of validity that it is true at this point. Let us call this point Γ. I first prove that **O**P is true at Γ. It follows then that **P**P has to be true. I prove that it is not.
The two facts sufficient to make the proof work are these.
26) If a point Γ has no access to any other point then any formula of the form **O**P is true at Γ.
Proof. This follows from the evaluation rule for **O**X and classical predicate logic. Let Γ be any point in any(not necessarily serial) frame. Quantifying over points in a frame the evaluation rule says that

[120] Naturally, a lot of what is said in the following wins from putting it in the general context of other systems of modal logic. I have chosen not to do so, and focus on the specific traits of SDL or D.

$(\Gamma \Vdash \mathbf{O}P) \leftrightarrow ((\forall x)((\Gamma \Re x) \rightarrow x \Vdash P))$. If we have $\forall x \neg(\Gamma \Re x)$ then we can prove $\Gamma \Vdash \mathbf{O}P$. Because if we did not have it we would have $\neg(\Gamma \Vdash \mathbf{O}P)$ and then we would also have $\neg\forall x((\Gamma \Re x) \rightarrow x \Vdash P)$. From this follows that there is a point Ω with $\Gamma \Re Q$ and $\neg(\Omega \Vdash P)$. But since we assumed that Γ did not have access to *any* point it is also not related to Ω. So we also have $\neg(\Gamma \Re Q)$ and a contradiction which means that we must have $\Gamma \Vdash \mathbf{O}P$.

The other fact we need is this:
27) If there is no point accessible from a given point then no formula of the form **P**P is true at the latter point. This means that $\neg$ **P**P at the latter point.
27) follows from the valuation rule of **P**X and classical predicate logic. I will not present a proof. Now we have our desired result. Because since there is no point accessible from the point Γ, $\Gamma \Vdash$ **O**P is true there because of 26). And since **O**P→**P**P is valid in the frame it follows that **P**P is true at Γ. But because of 27) $\neg$ **P**P is also true at Γ we have **P**P and $\neg$ **P**P is true at Γ, which is impossible. Thus the frame must be serial. This concludes the proof.

This result means that I could have taken **O**P→**P**P as a global assumption of the models I wanted to consider (true at any point in those models) and have the same outcome as placing the restriction on the relation $\Re$ that it is serial.

An analytic tableau proof system for SDL

I will now present a systematic proof procedure for formulas of SDL called an analytic tableau system. A formula with a proof is a theorem of SDL. Tableau methods are analytic because they proceed by dissecting formulas into smaller parts. They are contrasted with natural deduction and axiom systems, which are called synthetic, because they build the formula to be proved out of smaller parts. To prove a formula one negates that formula and applies rules to it. A tree unfolds and when a formula and its negation occur in a branch it is closed. When all branches are closed the tree is closed and the original non-negated formula is a theorem. In semantic terms a tree tries out all logical possibilities. If they all lead to a contradiction the original formula must be false which means that its negation must be true. If the tree is completed and there is an open branch the tree is open and a counter example to the formula to be proven can be derived from the open tree. A *derivation* of a formula from a set of formulas is when the premises and the negated conclusion together close a tree. There are other useful ways of using analytic tableau. Let us take a few examples. If a set of formulas is consistent its completed (or saturated) tree will be open. If a formula occurs in all branches of that tree it can be derived from the set (because the negation of it closes the tree). I often use trees in such extended ways. [121] Analytic tableaux are also called semantic tableaux (or tableaus) but the term can be misleading. Melvin Fitting writes,

> One cannot really develop semantic tableaus without a semantics behind it.[122]

However, analytic tableaux is just as much a pure proof-procedure as any other system of logic, because it proceeds using only syntactic conventions. Even though the procedure is usually motivated by semantic considerations, it is important never to confuse semantics with this procedure. (Fitting of course never confuses the two and writes brilliantly about analytic tableaus. The point was just to be a bit cautious with the term semantic tableau).[123] They are combined by soundness and completeness theorems.

Now I present the rules for SDL.

I use prefixed formulas. A prefixed formula is a formula of the form s X. Here s is a sequence of integers and X is a formula of the language we are already using. In the intended interpretation, the prefix denotes a point in a frame where the formula is true. I separate the terms (the integers) of the sequence with points, as in 1.1.1. In the following s is a sequence of integers and m is an integer. In the intended interpretation the minimal requirement of the point Γ denoted by a prefix s, is that for any prefix s.m (denoting a point Ω) in the same branch as s, we have $\Gamma\mathfrak{R}\Omega$. (Of course there may be no such prefix s.m in the branch).

In constructing counter models I will sometimes use the prefixes themselves as names of points in a frame and write about their relation as for example ($1\mathfrak{R}1.1.$) meaning the point denoted by 1 has access to the point denoted by 1.1.

Rules for Propositional SDL

Basic Rules

DN-rule	$\wedge$-rule	$\vee$-rule	$\rightarrow$-rule	$\leftrightarrow$-rule
$s \neg\neg\alpha$	$s \quad \alpha\wedge\beta$	$s \quad \alpha\vee\beta$	$s \quad \alpha\rightarrow\beta$	$s \quad \alpha\leftrightarrow\beta$
$s \quad \alpha$	$s \quad \alpha$	$\swarrow \quad \searrow$	$\swarrow \quad \searrow$	$\swarrow \quad \searrow$
	$s \quad \beta$	$s\ \alpha \qquad s\ \beta$	$s\ \neg\alpha \qquad s\ \beta$	$s\ \alpha \qquad s\ \neg\alpha$
				$s\ \beta \qquad s\ \neg\beta$

[121] See (Jeffrey, 1967), for extended use of trees, also a synthetic proof procedure using trees.
[122] (Fitting, 2002), p. xiv.
[123] This is why I use the proof-theoretic term “derivation” about a proof a formula from a set of formulas instead of the semantic “consequence”.

$\neg\wedge$-rule

s $\neg(\alpha\wedge\beta)$

s $\neg\alpha$ s $\neg\beta$

$\neg\vee$-rule

s $\neg(\alpha\vee\beta)$

s $\neg\alpha$

s $\neg\beta$

$\neg\rightarrow$-rule

s $\neg(\alpha\rightarrow\beta)$

s α

s $\neg\beta$

$\neg\leftrightarrow$-rule

s $\neg(\alpha\leftrightarrow\beta)$

s α s $\neg\alpha$

s $\neg\beta$ s β

To understand the deontic rules, which follow, note the following facts about relational semantics for SDL.(This is the kind of semantic motivation I was discussing above).

28) If **O**X is true at a point then at all points accessible from it X is true.

29) If $\neg$**O**X is true at a point then there is a point accessible from it where X is not true and hence $\neg$X is true at that point.

30) If **P**X is true then there is a point accessible from it where X is true.

31) If $\neg$**P**X is true at a point then at all points accessible from it $\neg$X is true.

These considerations lead to the following rules.

Deontic Rules

s   **O**α s $\neg$**P**α

s.m α s.m $\neg\alpha$

Where s.m is already in the branch.

s   **P**α s $\neg$**O**α

s.m α s.m $\neg\alpha$

Where s.m is new to that branch.

Further note as was proved above that if a frame is serial then **O**$\alpha\rightarrow$**P**α is valid, so if **O**α is true at any point then **P**α is true at that point by modus ponens. The contraposition of **O**$\alpha\rightarrow$**P**α, is $\neg$**P**$\alpha\rightarrow\neg$**O**α. They are equivalent by classical propositional logic and so both valid in the same frames. In SDL we have restricted ourselves to serial frames, because we want every obligation to be permitted. We do not want every obligation to be automatically fulfilled in the actual world so we do not take the step further to reflexivity (**O**P$\rightarrow$P). We get the following rules.

s $\mathbf{O}\alpha$ s $\neg\mathbf{P}\alpha$

s $\mathbf{P}\alpha$ s $\neg\mathbf{O}\alpha$

In the intended interpretation this rule ensures that given $\mathbf{O}\alpha$ there is always an accessible point where α is true, and similarly for $\neg\mathbf{P}\alpha$ there will be a point where $\neg\alpha$ is true.
When we apply the rules above to formulas we get trees. A node on a tree is a point where there is at least one prefixed formula. The origin is the topmost node. It has no nodes above it. A node has up to 2 successors immediately below it resulting from the application of a rule on a prefixed formula on or above that node. Endpoints are the bottom most nodes and they have no successors. A branch is a sequence of nodes starting somewhere and going down through the tree (trees are botanically upside-down), without ever going up again, always picking a successor of a node as the next term of the sequence. A maximal branch starts at the origin (the topmost node) and goes to an end point. If a prefixed formula and its negation occur in a branch it is closed. I mark this with an X underneath that branch. If all branches are closed the tree is closed. This proves the formula negated at the origin. If one branch in a tree is not closed the tree is open.

Completeness and Soundness

A system is sound with respect to its formal semantics if any provable formula (theorem) is valid. It is complete if every valid formula is provable. For the analytic tableau system soundness and completeness proofs can proceed by extending the semantic notion of a satisfiable set of formulas (not contradictory) to prefixed formulas and branches and through the notion of a completed tree. Here I am primarily interested in philosophical applications of systems so I will simply refer the reader to proofs.[124]

Proofs, Trees and Deontic Paradoxes

Let us turn to the application of SDL. I will start out by discussing some of the so-called deontic paradoxes. When a tree is not a proof I will simply call it a tree. Open trees are used to extract counter examples.

[124] E.g. (Fitting, Mendelsohn, 1998), pp. 57-65).

Ross´ Disjunctive Paradox

The two following proofs show that Ross´ disjunctive paradox can be derived in both of the forms discussed by Ross himself. From “You have to send the letter” we can infer both “you have to burn the letter or send the letter” and “you have to burn the letter or you have to send the letter”.

Q: You send the letter

P: You burn the letter

Proof 32.1)		Proof 32.2)	
1	$\mathbf{O}Q$	1	$\mathbf{O}Q$
1	$\neg\mathbf{O}(P\vee Q)$	1	$\neg(\mathbf{O}P\vee\mathbf{O}Q)$
1.1	$\neg(P\vee Q)$	1	$\neg\mathbf{O}Q$
1.1	$\neg P$		X
1.1	$\neg Q$		
1.1	Q		
	X		

A defence is that a disjunctive obligation does not imply a conjunctive or *choice-offering* permission as it sometimes seems to do in ordinary language. If I am told that I either have burn the letter or have to send it, it could be taken to mean that I choose which one to do. But in SDL we cannot derive $\mathbf{P}P\wedge\mathbf{P}Q$ from $\mathbf{O}P\vee\mathbf{O}Q$. $\mathbf{O}P\vee\mathbf{O}Q$ is sometimes called an *alternative-presenting* obligation.

Tree 33)

1 $\mathbf{O}P\vee\mathbf{O}Q$

1 $\neg(\mathbf{P}P\wedge\mathbf{P}Q)$

1 $\neg\mathbf{P}P$		1 $\neg\mathbf{P}Q$	
1 $\neg\mathbf{O}P$		1 $\neg\mathbf{O}Q$	
1 $\mathbf{O}P$	1 $\mathbf{O}Q$	1 $\mathbf{O}P$	1 $\mathbf{O}Q$
X	1.1 $\neg P$	1.2 $\neg Q$	X
	1.1 Q	1.2 P	

The open branches can be turned into counter models. We concentrate on the first open branch from the left. Let g be 1, 1.1. with $1\mathfrak{R}1.1$, $1.1\mathfrak{R}1.1$[125], and with $1.1 \Vdash Q$ and $1.1 \Vdash \neg P$. It follows that

[125] 1.1 being related to itself gives it a point related to it and makes the frame serial. The full formal definition of the frame $\langle g, \mathfrak{R}\rangle$ is $g=\{1, 1.1\}$ $\mathfrak{R}=\{\langle 1,1.1\rangle, \langle 1.1,1.1\rangle\}$. So I take the prefixes themselves to be the “worlds” of the frame.

$1 \Vdash \mathbf{O}Q$ and thus $1 \Vdash \mathbf{O}P \vee \mathbf{O}Q$. It also follows that $1 \Vdash \neg\mathbf{P}P$ thus *not*($1 \Vdash \mathbf{P}P \wedge \mathbf{P}Q$) and so $1 \Vdash \neg(\mathbf{P}P \wedge \mathbf{P}Q)$. This is also a counter model to $\mathbf{P}(P \wedge Q)$ from $\mathbf{O}P \vee \mathbf{O}Q$, since we have $1.1 \Vdash \neg(P \wedge Q)$ and so $1 \Vdash \neg\, \mathbf{P}(P \wedge Q)$. Nor can $\mathbf{P}(P \wedge Q)$ or $(\mathbf{P}P \wedge \mathbf{P}Q)$ be derived from $\mathbf{O}(P \vee Q)$ in SDL. Nor do any of the converses hold. I leave most of the remaining display of counter models to the interested reader. Here I will just show an interesting special case. A *free-choice* permission is of the form $(\mathbf{P}P \wedge \mathbf{P}\neg P)$. If P is that you smoke it means that you are permitted to smoke and permitted not to smoke. It is not possible to derive that either P or $\neg P$ are obligatory(that you have to smoke or not smoke) from this.

Tree 34)

1	$\mathbf{P}P \wedge \mathbf{P}\neg P$
1	$\neg(\mathbf{O}P \vee \mathbf{O}\neg P)$
1	$\neg\mathbf{O}P$
1	$\neg\mathbf{O}\neg P$
1	$\mathbf{P}P$
1	$\mathbf{P}\neg P$
1.1	P
1.2	$\neg P$

Tree 34) shows how $\mathbf{O}P$ and its contrary can both be false and how the subcontraries $\mathbf{P}P$ and $\mathbf{P}\neg P$ can both be true without any contradiction. This corresponds with the deontic square of opposition above. A formal semantic counter model has $1.1 \Vdash P$, $1.2 \Vdash \neg P$ and $1\mathfrak{R}1.1$, $1\mathfrak{R}1.2$, (plus $1.1\mathfrak{R}1.1$ and $1.2\ \mathfrak{R}1.2$ to make it serial). Both the permissions are true and both the obligations are false in this model, as the reader may verify.($\neg P$ is true at 1.2. so $\mathbf{P}\neg P$ is true at 1. Similarly for $\mathbf{P}P$ so $(\mathbf{P}P \wedge \mathbf{P}\neg P)$ is true at 1, etc.).

The same considerations go for the permissive variant of Ross´ paradox from $\mathbf{P}P$ to $\mathbf{P}(P \vee Q)$. It does not follow that P *and* Q are both permitted. The right formalisation of a *free choice permission* is $\mathbf{P}P \wedge \mathbf{P}Q$[126]. This concludes my discussion of Ross´ disjunctive paradox in the context of SDL.

Prior´s Paradox of Derived Obligation and Ross´ Conditional Fallacy

Arthur Prior presented his two paradoxes of derived obligation in (Prior, 1954). The most important of these is the formula $\neg\mathbf{P}Q \rightarrow \mathbf{O}(Q \rightarrow P)$[127], which is a theorem of MDL and hence

[126] Cf. (Føllesdahl, Hilpinen, 1971), p. 23: If "a is permitted to smoke or kill" is a free-choice permission, it should be formalized $\mathbf{P}P \wedge \mathbf{P}Q$…

[127] (Prior, 1954), p. 64. The other paradox is $\mathbf{O}Q \rightarrow \mathbf{O}(R \rightarrow Q)$. The parallel to the paradoxes of material implication of classical propositional logic, $R \rightarrow (Q \rightarrow R)$ and $R \rightarrow (\neg R \rightarrow Q)$ is obvious.

SDL.[128] Recall the definition of commitment given by von Wright as $\mathbf{O}(Q \rightarrow P)$, which he took to mean

> Doing one act commits us to do another act...[129]

With this interpretation Prior´s formula amounts to:

> …the doing of what is forbidden, commits us to the doing of anything, whatsoever.[130]

And this is naturally and undesirable conclusion. Here is a proof in SDL.

Proof 35)

1 $\neg\mathbf{P}Q$
1 $\neg\mathbf{O}(Q \rightarrow P)$
1.1 $\neg(Q \rightarrow P)$
1.1 Q
1.1 $\neg P$
1.1 $\neg Q$
X

A common thing to say at this point is that there is something wrong with SDL for allowing proofs like 35). Another way out is to question the formalisation of conditional obligations as $\mathbf{O}(P \rightarrow Q)$. There is an alternative formalisation of conditional obligations in SDL: $P \rightarrow \mathbf{O}Q$. This seems better, because the ordinary language antecedent of the first premise in 35) is in fact a proposition, and we are now symbolising it as such. With this formalisation Prior´s paradox aborts as the reader can check. It is a small but interesting point that the alternative formalisation is not available in MDL as $Q \rightarrow \mathbf{O}P$ is not well formed in MDL.

Ross´ Conditional Fallacy

There is another reason to adopt the alternative formalisation of commitment, which often goes unnoticed. Consider the following instance of Ross´ conditional fallacy.

36) If you are going out you have to take an umbrella.
You have to go out
____*
You have to take an umbrella

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129 (Von Wright, 1957), p. 61.

130 (Prior, 1954), p. 64.

We have discussed above why this "inference" is a fallacy. The antecedent of the implication is not a normative so modus ponens does not hold in case the second premise is not fulfilled. Only in the case where you actually are going out you have to take an umbrella. If you violate the second premise you should not be required to take an umbrella. In other words we should be able to formalise the premises and the negation of the conclusion without a contradiction. We would expect that the set of premises are impossible to fulfil but not that they are logically inconsistent.
Let us try formalising the first premise of 36) as Von Wright would have it in MDL.

Q: You are going out

R: You take an umbrella

Proof 36.1)

```
     1   O(Q→R)
     1   OQ
     1  ¬OR
     1.1 ¬R
     1.1  Q
     1.1  Q→R
    /          \
1.1  ¬Q       1.1  R
      X              X
```

The result is a closed tree, which validates the fallacy. On the other hand if we formalise the first premise of 36 as Q→**O**R we get the following tree.

Tree 36.2)

```
     1   Q→OR
     1   OQ
     1  ¬OR
   /          \
1  ¬Q        1   OR
1.1 Q             X
```

This tree does not close, and this is as it should be. A counter model is this: 1 ╟¬Q, 1.1 ╟ Q, 1ℜ1.1, 1.1ℜ1.1.. In this model **O**Q is true at 1 and Q is not. This is the case where an obligation is in force but is violated. In a perfect world it is fulfilled but not in this one, which is not contradictory. Von Wright did not have the possibility of formalising conditional obligations this way as mentioned above. But in SDL it is, and it is generally the right way to formalise conditional obligations. In cases like "If you have to go out, you have to take an umbrella", where the antecedent is an obligation too it is correct to symbolise (**O**P→**O**Q), but it is important to note these subtle differences. Finally, let us follow (Nute, Yu,1997), and call the derivation of **O**Q from **O**(R→Q) and **O**R for *deontic detachment* and the derivation of **O**Q from R→**O**Q and R for *factual detachment.* [131]

Chisholm´s Contrary-to-Duty Imperatives

A contrary-to-duty obligation or a secondary obligation is one, which is activated when somebody has violated a primary obligation. You are not supposed to hurt anybody, but if you do, you should apologise, for example. If you are not going to attend a meeting you should at least tell the people you are supposed to meet, and so on. Secondary obligations usually neither cancel primary obligations nor automatically "make everything all right". They are often presented as attempts to remedy things that have already gone wrong or to show the next best thing to do if things are not expected to go optimally. As such contrary-to-duty obligations are an important part of ordinary life and language and should not be formally contradictory. Roderick Chisholm´s contrary-to-duty paradoxes show that they do lead to paradoxes in a strict sense of the word in that they imply a contradiction.[132] Again this result hinges on von Wright´s formalisation of conditional obligations as **O**(P→Q). Chisholm´s own example was the following: [133]

37) You ought to go help your neighbours!
You ought to call them if you do go.
If you do not go you ought to not call them.
You do not go

Let
Q: you go help your neighbours
P: You call your neighbours
If the second premise is formalised as **O**(Q→P) and the third as ¬Q→**O**¬P the paradox will follow from **O**Q and ¬Q, but as we have seen above **O**(Q→P) is probably not the right way to formalise conditional obligations. Also, it seems obvious that we can articulate the third and fourth premise together as "you ought to call them if and only if you do go". Then we must formalise the two conditionals the same way.

[131] (Nute,1997), pp. 6-7.
[132] (Chisholm, 1964).
[133] (Chisholm, 1964), pp. 34-35.

37.1)

$1 \quad \mathbf{O}Q$

$1\ Q\rightarrow \mathbf{O}P$

$1\ \neg Q\rightarrow \mathbf{O}\neg P$

$1\ \neg Q$

$1\ \neg\neg Q \qquad\qquad 1\ \mathbf{O}\neg P$

X

$1\ \neg Q \qquad 1\ \mathbf{O}P$

$1\ \mathbf{P}\neg P \qquad 1\ \mathbf{P}\neg P$

$1.1\ \neg P \qquad 1.1\ \neg P$

$1.1\ Q \qquad 1.1\ P$

X

The tree is open so there is no paradox. An informal counterexample is this. You ought to go see your neighbours. You do not go. Then you ought to not call them. Then in a perfect world you do not call them. Against this formalisation Føllesdal and Hilpinen wrote:

> If(the formalisation) is modified in accordance with this proposal the contradiction is avoided, but then we get paradoxes which resemble the paradoxes of commitment discussed above.[134]

The authors refer to an article by Bengt Hansson for clarification. Here we find that though in the ordinary language variant 37) the second premise is not felt to follow from the fourth, in the formal version $Q\rightarrow\mathbf{O}P$ follows from $\neg Q$.[135] First, these are neither contraries or contradictories. Secondly, this is one of the paradoxes of material implication for classical propositional logic. So at least the formalisation in 37.1) reduces part of the problem to that problem which seems better than a contradiction to me. Q an $\neg Q$ are impossible together so anything follows from them. A more serious part of the problem is that in "the perfect world" 1.1 you go help your neighbours even though you also do not call them. This is a problem for standard deontic logic. In the perfect world all obligations are fulfilled, secondary and primary. If we propose a reinterpretation of perfect worlds as points of fulfilment, the problem can be clearly stated: If an obligation is violated, then there should not be a point of fulfilment for it in a model. But there is, because *all* obligations are fulfilled in perfect worlds. So, this reinterpretation of SDL in terms of fulfilment is simply not possible. We want a deontic logic to give us information about which of our primary obligations are fulfilled *and* which are violated, and which of our secondary obligations are fulfilled and which are

[134] (Hilpinen (ed.), 1971), p. 25.
[135] (Hansson, 1970), p.132-133.

violated and possibly so on for tertiary obligations. SDL cannot do this and cannot easily be adapted to do this. It is the purpose of the system developed in the second part of this thesis to provide this kind of information. [136]

Other Paradoxes

There are other paradoxes in standard deontic logic. Some are formulated for standard deontic logic some for other systems. They usually show that the problems with the conditional are so stubborn that they just seem to follow along into dyadic, temporal and quantificational versions of deontic logic. We shall look at two examples below, which will not be solved for SDL by the method of formalisation above. Before turning to other logics, consider the following transitivity rules for deontic logic. They say that if an obligation is true in the actual world it is true at any perfect world.

Deontic Transitivity Rules

s **O**P s ¬**P**P

s.n **O**P s.n ¬**P**P

Where s.n is already in the branch.

Adding this rule to SDL gives SDL4 or D4.[137] The semantic counterpart to the rule is making the accessibility relation transitive, thus imposing a restriction to serial, transitive frames. The extra restriction is this. For any Δ, Ω, Γ in a given frame:

38) If $\Gamma\mathfrak{R}\Delta$ and $\Delta\mathfrak{R}\Omega$ then $\Gamma\mathfrak{R}\Omega$. (transitivity)

Adding it could be justified this way. If an obligation holds in the actual world and is fulfilled in a perfect world, then it must also be obligatory to fulfil it in that perfect world. Because if it was not obligatory why would it be fulfilled there? This type of justification relies on the perfect world idea, but we have already seen how this idea may be flawed above.

Alternative Deontic Logics

The term alternative deontic logics I take to cover deontic logics, which deviate from standard deontic logic. These are numerous and my presentation here will remain selective and incomplete.

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136 For instance, if we suggest that violated obligations are not fulfilled in deontic alternatives by blocking their access, we get a problem that we also have to block all logically equivalent formulas, and our rules cannot do this in a simple way. Even if this problem was solved, the evaluation rules will not work anymore as a consequence, unless we want an obligation to loose force because it is violated (which we do not!). As it is right now an obligation is true iff it is fulfilled in *all* deontic alternatives.

Most trends of research(except to some extent quantified SDL) I will only treat in general terms, if I treat them at all. Some of them can be considered extension of standard deontic logic. This is the case for quantified SDL. Most are systems in their own right. It is thus the purpose of this paragraph to briefly indicate some of the alternative roads taken in the literature, which I will not be able to go in depth with.

Dyadic Deontic Logic

Dyadic deontic logic was first introduced in (von Wright, 1956) and developed by Rescher and others. It introduces a special sign for the conditional, meant to cover conditional obligations. An obligation is usually written **O**(Q /R) , which is read" Q is obligatory, given R". However **O**(Q/Q) is a theorem of many dyadic logics. It could be read two ways. If we read **O**(Q/Q) as expressing conditional obligations of factual detachment this is absurd, because then everything which is, is obligatory. Most dyadic deontic logics must thus be taken to be deontic detachment systems, and then **O**(Q/Q) is not a problem. If you have to do Q then you have to do Q, is not peculiar. However, now the problem is that there is no way to formalise the most common conditional obligations in dyadic logics, because these are indeed the ones that require factual detachment as shown above. Another way to go is to block deontic detachment and allow factual detachment. Nute and Yu discuss systems by Mott, (1973) and others. Here the problem is that there is no way to formalise *deontic detachment*.

Nute and Yu write:

> It has become increasingly clear that an adequate system of deontic logic should be able to allow both deontic detachment and factual detachment yet to handle them coherently.[138]

There are many dyadic, deontic logics. In fact the little I have said does not do them any justice. I can only encourage the reader to investigate some of them on her or his own.

Temporal Deontic Logic

Temporal deontic logics have been developed, e.g. by Richard Thomason in (Thomason, 1981a) and J. A. van Eck(Van Eck, 1981). In the systems obligations are true at different points of time. A

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[137] It is called 4 for a historical reason, which is not important here.

[138] (Nute (ed.), 1997), p. 7.

secondary obligation may incur at a later time. Forrester´s *gentle murder paradox*[139] is usually seen as a blow to temporal deontic logics.[140]

The Gentle Murder Paradox

39) It is obligatory that you do not murder X.
If you murder X, then it is obligatory that you do it gently.
You do murder X
Murdering X gently entails murdering X.

I will present it for SDL and explain why distinguishing times will not work.

M: You murder X.

G: You murder X gently.

That murdering gently entails murdering we represent by taking G→M as a global assumption.[141] This means that we can add the sentence to any prefix occurring in the tree.

Proof 39.1)

```
             1   O¬M
             1   M→OG
             1   M

   1 ¬M           1   OG
     X            1   PG
                  1.1 G
                  1.1 ¬M
                  1.1 G→M

                ¬G        M
                 X        X
```

In 39.1) the global assumption is just added once, at 1.1 because we do not need it at point 1. It is important to see that this paradox is also a real blow to SDL, since the tree closes contrary to intuition. Intuitively the premises are not inconsistent. Yet the tree closes. It is from a family of paradoxes knows as *good-Samaritan paradoxes.* Other variants are called the paradox of the

139 (Forrester, 1984)
140 See (Nute (ed.), 1997), pp. 4-5.

knower and the good-Samaritan paradox. These older versions could be questioned because they involved odd presuppositions (The good-Samaritan. Helping a man you will murder a week from now presupposes that you will murder a man a week from now, it does not entail it, and furthermore how can it ever be certain that you will murder him?), and principles from epistemic logic (the paradox of the knower). The gentle murder paradox took these paradoxes to a new level, because if you murder somebody gently it seems to entail that you murder somebody without a doubt. The solution of formalising conditional obligations the alternative way is not sufficient to avoid the paradox and here follows the reason why it will not help to distinguish different obligations at different points of time. The solution of using a temporal deontic logic does not work because the secondary obligation to murder gently does not come after the one not to murder, but simultaneously with it. The agent is murdering at the same time as he or she is murdering gently.

> For timeless CTD structures, such as the gentle murderer… the use of temporal deontic logic does not help, since all statements pertain to the same point of time.[142]

A CTD structure is a contrary-to-duty structure. In conclusion, distinguishing obligations at separate points of times will not work for this paradox.

It seems to me that the problem for SDL with regard to this paradox is the same as with the counter-intuitive result of the Chisholm paradox above. The violated obligation is carried into the perfect world.

For recent research developing Thomason´s work on indeterministic time and combining it with decision perspectives for agents see (Horty, 2001).

Deontic Preference Logics

Another trend of research combines preference logic with deontic logic. It started with (Hansson,1970), which combined von Wright´s work on deontic logic and preference or axiological logic.[143] It was developed further in (Lewis, 1973). The idea is to put a preference relation on deontic alternatives. Of the accessible deontic worlds some are regarded better than others, from the point of view of the moral agent or the norm giver. See e.g. (Hansson, 1997) for work in this area. Many defeasible deontic logics are also based on some kind of preference semantics.[144]

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[141] As an alternative to adding specific alethic modality rules.
[142] Prakken, Henry, and Sergot, Marek. (1997). Dyadic Deontic Logic And Contrary-To-Duty Obligations. (Nute (ed.), 1997), p. 244.
[143] See (Hilpinen, 1970), pp. 121-147.
[144] See (Nute, Xiaochang 1997).

Hector-Neri Castañeda´s Logic

Hector-Neri Castañeda developed several deontic logics. They all seem to be based on a basic distinction between practical and descriptive language, and so lead to two-sorted languages. He presented data, called *crucial linguistic data* to substantiate the claim about a fundamental divide between practical and descriptive language. The most extensive treatment is given in (Castañeda, 1975). (Castañeda, 1981) gives a concise overview. He illustrates his claim with examples such as the following:

> Alchourrón is obliged (obligated required) by his work agreement with Bulygin to do the following: RE-WRITE THE FOOTNOTES, if and only if he mislays them, *AND* SEND THE FOOTNOTES TO CARRIO, only if *he re-writes them.*[145]

The clauses written with capital letters called *practitions* he considers deontic foci, which means that they can be "bound" by the deontic operators, or that it is these an deontic operator operates on.. He describes a practition as:

> …an *action deontically considered as the focus of obligatoriness…*[146]

Practitions can be expressed in ordinary languages by imperatives (also called mandates), and by infinitive or subjunctive clauses as in the sentence above. It is not the mandates but the practitions, which have logical relations between them. Practitions have no truth-value, but are two-valued. He says that a practition P for an agent Y to do A can be expressed: X to do A. Between practitions of this type logical relations are supposed to hold. It presents a problem to my intuition that clauses like P are not grammatical sentences of English, because sentential logic is supposed to work between sentences. Deontic sentences, on the other hand, created by applying a deontic operator on a practition, are regarded propositions with a truth-value. Here I will treat only Castañeda´s solution to Good Samaritan paradoxes such as the Gentle Murder above. He sees the origin of the paradox in the following principle of inheritance, which we met already as 1) by Mally above:

> (P) If X´s doing A entails Y´s doing B, then that X´s doing A is ___obligatory entails that Y´s doing B is ___obligatory.[147]

He proposes to replace it with

> (P*) If practition X to do A entails Practition Y to do B, then It is obligory$_i$ that X do A entails It is obligatory that Y do B.[148]

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[145] Castañeda, Hector-Neri. (1981). The paradoxes of Deontic Logic: The Simplest Solution to all of them in one Fell Swoop". (Hilpinen,1981), p. 42.

[146] (Hilpinen (ed.), 1981), p.42.

[147](Hilpinen (ed.), 1981), p. 52. The lines before obligatory are blanks where a specific type of adverbial qualifier can be inserted, for instance "morally" or "legally".

[148](Hilpinen (ed.), 1981), p.53.

He presents the paradox:

40) Arthur is legally(morally) obligated TO BANDAGE the man, which he will murder a week hence.
41) Arthur bandaging the man entails murdering the man a week hence.[149]

Even though 1) presupposes that Arthur will murder a man in a week, the principle P* will not lead to a paradox, because "which he will murder a week hence" is not considered practically.

The Fence Paradox

The following variant of the good-Samaritan paradox by Prakken and Sergot is a counter example to Castañeda´s idea that deontic foci are always *actions*.

42) There must be no fence around a cottage.
If there is a fence it must be white
There is a white fence entails there is a fence.[150]

Clearly, this is a set of possible normatives. But there are no actions involved. Prakken and Sergot write:

> Neither do solutions apply where the condition of a conditional obligation is regarded as a state of affairs and the content of an obligation as an act((Castañeda, 1981;…(…)…there are variants of the example where both are states of affairs; a holiday cottage regulation could say: there must be no fence around a cottage, but if there is a fence, it must be a white fence.[151]

Whether this is a counter example to Castañeda´s method, or it just means that we need a broader definition of practition is debatable.
The following seems to me another objection, at least to Castañeda´s specific method of bringing out the sentences, which are "practically considered" as opposed to the ones that are circumstances.

43) The following is obligatory for Andrea: If she has to take a bath, TO TAKE OFF HER CLOTHES.
44) The following is obligatory for Andrea: If she does not have to take a bath, NOT TO TAKE OFF HER CLOTHES.

[149] (Hilpinen (ed.), 1981), p. 53.
[150] See e.g. (Nute (ed.), 1997), p. 100.
[151] (Prakken and Sergot, 1997) in (Nute (ed.), 1997), p. 223. They first presented the paradox in 1994.

45) The following is obligatory for Andrea: NOT TO TAKE A BATH.

It seems clear to me that 45) and 44) entail that it is obligatory for Andrea not to take a bath with modus ponens because 45) is equivalent to the antecedent of 44). In SDL this example would require deontic detachment as defined above. But the method to bring out practitions fails in this case. In fact the conditions (or following Castañeda, circumstances) are normatives. Castañeda does not allow iterated deontic operators, so this does not present a solution to the problem. The grammatical "test" presented by Castañeda, does not bring out, that "she has to take a bath" is a normative. Given the care I have taken above to distinguish this type of example involving two normatives I am positive 43)-45) will not be confused with an instance of Ross´ Conditional fallacy which involves mixed sentences and factual detachment. In mixed cases Castañeda´s method generally works, as far as I can see. In cases of pure normative implications it fails.

Jaakko Hintikka´s Quantified Deontic Logic

Jaakko Hintikka has argued (1970) that deontic logic needs quantificational resources.

> The introduction of quantifiers is in fact already necessary for the purpose of analyzing adequately some of the fundamental concepts...[152]

and,

> Hence the relation of permissions to obligations…cannot be explained satisfactorily without quantifying words like "every" and "some".[153]

Apart from the fact, that Hintikka introduces a quantified deontic logic, the other crucial aspect of his approach is the use of possible world semantics. A very important feature of this is the method of proof Hintikka invented, which is essentially the same as the one displayed in the section about SDL above. Even though Hintikka´s semantics deviates a bit from the Kripke semantics that I have presented it is usually regarded as being similar to it. In fact it is easy to characterise Hintikka´s Deontic Logic (HDL) using the terminology of the previous section. The main difference between SDL_4 and HDL is in what Hintikka calls the deontic alternatives, which are indeed "perfect worlds" in the respect that all obligations are fulfilled in them and all prohibitions are respected.[154] He compares this to Immanuel Kant´s *Kingdom of Ends,* which would be realized,

[152] Hintikka, Jaakko. (1971) Some Main Problems of Deontic Logic. In (Hilpinen,1971), p.62.
[153] (Hintikka, 1971). In (Hilpinen (ed.), 1971), p. 65
[154] That there are several deontic alternatives is made plausible by the fact that different permissions in a possible world may not be used in the same or in all or in one of the deontic alternatives.

if all the maxims based on the categorical imperative *were followed without exception.*[155]

This result Hintikka gets through adding reflexivity for the relation $\mathfrak{R}$, (axiomatically $\mathbf{O}P \rightarrow P$) but *only* in perfect worlds.[156]

HDL is a quantified SDL_4 with the rule added which in Hintikka´s own terminology is articulated as follows:

$(C.O)_{rest}$ If $Op \in \mu^*$ and if μ^* is a deontic alternative to some model set μ, then $p \in \mu^*$.[157]

A *model set* is a possible world. The crucial case is when μ is the actual world (or another "naturalistic" possible world), where reflexivity does *not* obtain, because the actual world is not a deontic alternative to any world, or alternatively, if it is, we immediately have reflexivity for obligations in the actual world, which is very unlikely. In the terms we have used so far the semantic restriction we need is the following, which I call relative reflexivity. For any points Γ and Ω in a frame:

46) If $\Gamma\mathfrak{R}\Omega$ then $\Omega\mathfrak{R}\Omega$. (relative reflexivity).

Adding this restriction on $\mathfrak{R}$ confines us to frames which are serial, transitive and relatively reflexive.

In the terminology from the previous section we define the rule T*rel.* for HDL

T*rel.*: s.n $\mathbf{O}P$ s.n $\neg\mathbf{P}P$

s.n P s.n $\neg P$

Where s is already in the branch.

As Hintikka points out, T*rel.* and the transitivity rule 4 together imply the first deontic rule, because obligations are transferred to deontic alternatives by the 4 rule and fulfilled there by the T*rel.* rule, and the first deontic rule makes sure that obligations are fulfilled in deontic alternatives.[158] The main difference is with iterated operators. To see what T*rel.* adds to deontic logic the following is instructive, and it brings us back to Ross´ conditional fallacy once again.

Deontic Consequence and Logical Consequence

Hintikka claims that we often confuse the schema

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155 (Hintikka, 1971).(Hilpinen (ed.), 1971), p. 73.

156 The three salient *differences* between Hintikka and Kant, the former describes as follows, p.74: "...our concept of a deontic alternative does not contain any reference to a particular moral principle"(...) "It is a relativized notion , for it refers to the possible world which a deontic alternative is alternative to"(...)...a deontic alternative is not a unique entity, contrary to the way in which Kant seems to have looked upon his "Kingdom of Ends".

157 (Hintikka, 1971). (Hilpinen (ed.), 1971), p.71.

47) $(\mathbf{O}P\wedge(P\rightarrow\mathbf{O}Q)) \rightarrow \mathbf{O}Q$[159]

with

48) $\mathbf{O}((\mathbf{O}P\wedge(P\rightarrow\mathbf{O}Q)) \rightarrow \mathbf{O}Q)$[160]

47) is Ross conditional fallacy for deontic logic, and it is not valid in Hintikka´s system or in SDL as we have seen. That it is not Hintikka (rightly) sees as an indication that:

> …many of our obligations in fact remain unfulfilled[161]

A factual conditional obligation and an obligation to the antecedent do not make the antecedent true, only required. The obligation to the antecedent may be violated. 48), on the other hand, essentially *transfers the reasoning to a perfect world* in Hintikka´s system. This is a quite brilliant solution. Since in a perfect world, all obligations are fulfilled, 48) is valid in HDL (but not in SDL). Hintikka calls reasoning in perfect worlds for deontic consequence and reasoning in a "normal" possible world for logical consequence. It is, he says, the difference between $P\rightarrow Q$ being valid and $\mathbf{O}(P\rightarrow Q)$ being valid.[162] To me, though, it seems like the difference between 47) being true at a point or not. In a perfect world where all obligations are fulfilled 47) is true. In a "naturalistic" possible world it is not. This is what HDL succeeds in achieving, in contrast to SDL.

That 48) is not valid in SDL or SDL_4 is because we do not have relative reflexivity as a constraint there. 48) just moves 47) one step further from the actual world in SDL and in SDL_4 48) is itself also moved there.[163]

So far I have simply described a propositional version of Hintikka´s deontic logic. Now I turn to quantification of standard deontic logic. In the following paragraph I will discuss Hintikka´s claim that quantification of deontic logic is necessary and contrast his account with Hector-Neri Castañeda´s claim that ordinary deontic English is completely extensional. [164]

Quantified Standard Deontic Logic

First some remarks about quantification and modal logic in general, and about a limitation of my presentation. The limitation is that I am going to concentrate on first-order deontic logic. Higher order modal logics (also called intensional logics) are very interesting, but I had to stop

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[158] (Hintikka, 1971). (Hilpinen (ed.), 1971), p. 71.
[159] (Hintikka, 1971). (Hilpinen (ed.), 1971), pp.77-87.
(Hintikka, 1971). (Hilpinen (ed.), 1971), p. 79.
[161] (Hintikka, 1971). (Hilpinen (ed.), 1971), p. 80.
[162] (Hintikka, 1971). (Hilpinen (ed.), 1971), p.78..
[163] Some comments on Hintikka´s logic: The actual world cannot be a deontic alternative to any world on pain of Ross´ fallacy. Iterated obligations can be reduced to one in deontic alternatives, where the T-rule works. Different permissions create different deontic alternatives to a possible world.

somewhere.[165] In first-order SDL we add a set of individuals to our frames, which is to serve as the domain of the quantifiers. There are two ways of doing this. If we allow the domain to change from world to world we get *varying domain semantics*. With this, at a point Γ and for a predicate Q and an object x we may have Qx at Γ even though x is not in the *domain* of Γ. An example from alethic modal logic is the following sentence: "Peter Parker is Spiderman." Even though none of the two individuals considered exist in the actual world, this could still be considered a true identity statement in our world. In another possible world "Peter Parker is not Spiderman" is true, but this is not the actual world. According to the mythology of superhero comics adhered to in our world (after the invention of the character), this is a true identity statement. It is a limitation of classical first-order logic that such distinctions cannot be expressed, and a limitation of philosophers adhering dogmatically to first-order classical logic to call such distinctions "meaningless".

Corresponding to varying domain semantics is *actualist quantification*. This means that the quantifiers quantify over the individuals at a single point. We only quantify over what actually exists at a point.

The other way of doing first-order modal logic is with *constant domain semantics.* Now the domain of individuals stays the same throughout a model. Corresponding to this is *possibilist quantification*. The quantifiers now also quantify over what possibly exists at a point. With this interpretation the individuals of constant domain semantics are regarded possible individuals. They might be turned into actual individuals at a point through adding an existence predicate to the language, and in this way actualist quantification can be emulated in constant domain semantics. It is also possible the other way around, even though this is more complicated. The result is that it turns out that the two forms of quantification are different manners of speech, and that we can express the same philosophical ideas either way. I will present varying domain semantics. In doing this I follow the lead of Jaakko Hintikka (Cf. section below).

First-order Language

We use the same connectives and deontic operators as in propositional SDL but add two quantifiers ∀ (for all) and ∃ (there is). We have an infinite list of variables, $x_1, x_2, \ldots$(usually represented with letters from the end of the alphabet x, y,z ..) and infinite lists of relations symbols, $P_1{}^1$, $P^1{}_2, \ldots$ and

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164 (Castañeda, 1981).

165 See (Fitting, 2002), about higher-order modal logic.

$P^2_1, P^2_2, \ldots$ and so on where in P^n_m, n is called the *arity* of P. I will usually represent these P, Q, R and let the arity be clear from the number of variables in an expression.

First-order Atomic Formula

49.1) An atomic formula is any expression $R(x_1, \ldots, x_n)$. R is a relation symbol of arity n and x_1 to x_n are variables.

First-order Deontic Formula

The set of first-order deontic formula is the ones formed by replacing first-order atomic formula for propositional variable in the formation rules for propositional deontic logic. That is, any atomic first order formula is a formula, and so are the ones created by propositional deontic formation rules out of formulas. Furthermore the following:

49.2) If X is a formula and v a variable then $(\forall v)X$ and $(\exists v)X$ are formulas.

The variable v is said to be *bound* in X. If a variable is not bound in a formula X it is free. A formula where all variables are bound is called *a sentence* or a *closed formula*.

Castañeda and Hintikka on Quantified Deontic English

Before turning to formal semantics an informal discussion of quantified deontic logic. In (Castañeda, 1981) he claimed that quantification in ordinary deontic English is extensional. The reasons he considered were:

> 1. English deontic operators with the same adverbial qualifier do not iterate;
> 2. English deontic operators do not create referential opacity;
> 3. English deontic operators do not create a breakdown of identity;
> 4. English deontic operators do not create by themselves variations in the domain of quantification.[166]

He writes:

> Real obligations belong, naturally, to real agents and demand real actions on real objects and patients.[167]

Some of Castañeda´s claims may boil down to a claim that there is not a distinction corresponding to the *de re/de dicto* distinction in deontic English. For alethic modal logic this distinction is a scope distinction, which can be illustrated with an example. Take P to be "exists". $\Box(\exists x)P(x)$ means that in any possible world there is something which exists, which is a valid claim if we only work with non-empty domains. On the other hand $(\exists x)\Box P(x)$, means that there is something which exists in any possible world, which is invalid with varying domains, since it not a requirement that any

[166] (Castañeda, 1981). (Hilpinen (ed.), 1981), p. 66.

object is at all points. The latter sentence is necessity *de re* (about an object) the first is *de dicto* (about a sentence). We may take Castañeda to hold that such distinctions make no difference in deontic English. Castañeda claims that,

> …deontic operators do not in any way affect the range of quantifiers.[168]

The de re/de dicto distinction is sometimes shown by the difference between the two sentences:

50) It is necessary that something exists (de dicto).

51) It is necessary for something to exist (de re).

According to Castañeda the corresponding deontic sentences are equivalent.

52) It is obligatory for somebody to X

53) It is obligatory that somebody X.

> Now, there is no such ambiguity in the case of…"it is obligatory$_i$ that x do A"…from which…we can derive "it is obligatory$_i$ for someone to do A".[169]

Let us turn to Hintikka, who made the exact opposite claim that deontic English can never be understood properly without taking such scope-distinctions as between 52) and 53) into consideration. Now, in HDL we quantify over act-individuals in action-formulas such as Sx , read "x is an act of stealing". $(\exists x)S(x)$ is then read "There is an act of stealing". In HDL Hintikka reduces himself to "impersonal norms" which I make sense of by taking it to mean that they apply to everybody.[170] Alternatively we may think of them as applying to one of the same person. Let us look at an example of Hintikka´s, which happens to be the converse of the example discussed by Castañeda:

> $(\exists x)\ \mathbf{O}P(x) \rightarrow \mathbf{O}(\exists x)P(x)$
>
> This is obviously unacceptable intuitively as a logically valid principle. If there is, under the actual course of events, an act that ought to be an instance of forgiving a trespass, it clearly does not follow that there ought to be, under any deontically perfect course of events, an act of forgiving – and hence, presumably, also another earlier act of trespassing.[171]

Furthermore, Hintikka argues that $(\forall x)\mathbf{O}\neg P(x)$ should not imply $\mathbf{O}(\forall x)\neg P(x)$ and says it would correspond to moving over individual acts from the actual world to a perfect world.[172] This would mean restricting ourselves at least to what is called *monotonic frames*, anything at a point Γ exists at

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[167] (Castañeda, 1981). (Hilpinen (ed.), 1981), p. 67.
[168] (Castañeda, 1981). (Hilpinen (ed.), 1981), p. 67.
[169] (Castañeda, 1981). (Hilpinen (ed.), 1981), p. 67.
[170] (Hintikka, 1971). (Hilpinen (ed.), 1971), p. 60.
[171] (Hintikka, 1971). (Hilpinen (ed.), 1971), p. 102.
[172] (Hintikka, 1971). (Hilpinen (ed.), 1971), p. 63 and p. 103.

any point Ω accessible from Γ. The domain of Γ is a subset of Ω´s. Hintikka does not think this a good idea for similar reasons as in the example with the existential quantifier above. So he argues for varying domain semantics for quantified deontic logic.

Critical Commentary

To me, it seems like we should not want to restrict the quantifiers to specific objects be it agents (or acts). We want to be able to express both "Somebody has to make me a cup of coffee and "Somebody has to make me something to eat", where the last "something" refers to food-stuffs. Unlike Castañeda, we do also not want to restrict ourselves to existing objects. The "Something to eat," which is made, may not exist now. And in the sentence "You have to write a poem", there may not be any poems in the world but you have to write one. I think Hintikka´s example shows that there are significant scope distinctions involving quantifiers in English.

If we want an example involving agents take somebody who has committed a criminal offence in 2004. According to the law it is now true that

54) $(\exists x)\mathbf{O}P(x)$(de re)

It is obligatory *for somebody to be sent to jail in 2004.* On the other hand it is not according to the law obligatory *that somebody is sent to jail in 2004.*

55) $\mathbf{O}(\exists x)P(x)$(de dicto)

There could be no crimes committed in 2004 and so no one should be send to jail. If 55) were true, it would mean that the law required a person to have the official duty to go to jail in 2004, no matter if any crimes were committed or not. We might call therefore call this example, *the scapegoat example.* This subtle distinction is important.

Formal Semantics

An augmented frame F is a triple $\langle g, \mathfrak{R}, D\rangle$ where $\langle g, \mathfrak{R}\rangle$ is a frame and D is a function from g to non-empty sets. Each point thus has a non-empty domain associated with it that the quantifiers will quantify over. The domain of a point Γ is the set associated with that point, and we write it $D(\Gamma)$. The *domain of the frame* is the union set of the domains of all points and we write it D(F) .[173] An interpretation l assigns to each relation symbol R of arity n and to each $\Gamma \in g$ an n-place relation on the domain of the frame D(F). What this means is that each n-tuple of members of D(F), $\langle d_1, d_2, \ldots,$

[173](Fitting, Mendelsohn, 1998), p. 102.

$d_n\rangle$ is either in the relation or not. We write it $\langle d_1, d_2, \ldots, d_n\rangle \in l(R,\Gamma)$ if the d s are in the relation. Relations thus hold relative to points, but domains of relations are not restricted to points. What this means is that something can be in a relation at a point without being in the domain of that point. A *first-order model* is a structure $M = \langle g, \mathfrak{R}, D, l\rangle$. We will speak of the domain of the model, which is the same as the domain of the underlying frame. We write it $D(M)$. We restrict ourselves to serial frames and so to serial models. A *valuation* is a mapping v that assigns to each free variable x some $v(x) \in D(M)$. Let Γ be a point and v and w be two valuations. w is called an *x-variant at Γ* of v if they agree on all variables except possibly the variable x where $w(x) \in D(\Gamma)$. $M, \Gamma \Vdash_v X$, means that X is true at Γ under valuation v in the model M.

Truth in a First-order Model

For each point $\Gamma \in g$ and for each valuation v in a model M:

56.1) If R is a relation symbol of arity n, $M,\Gamma \Vdash_v R(x_1, \ldots x_n)$ if $(\langle v(x_1), \ldots, v(x_n)\rangle) \in l(R,\Gamma)$

56.2) $M,\Gamma \Vdash_v \neg X$ iff *not*($M, \Gamma \Vdash_v X$).

56.3) $M, \Gamma \Vdash_v (X \wedge Y)$ iff $M,\Gamma \Vdash_v X$ and $M, \Gamma \Vdash_v Y$.

56.4) $M, \Gamma \Vdash_v (X \vee Y)$ iff $M,\Gamma \Vdash_v X$ or $M, \Gamma \Vdash_v Y$.

56.5) $M, \Gamma \Vdash_v (X \rightarrow Y)$ iff, if $M,\Gamma \Vdash_v X$ then $M, \Gamma \Vdash_v Y$.

56.6) $M, \Gamma \Vdash_v (X \leftrightarrow Y)$ iff, $M, \Gamma \Vdash_v (X \rightarrow Y)$ and $M, \Gamma \Vdash_v (Y \rightarrow X)$.

56.7) $M,\Gamma \Vdash_v \mathbf{O}X$ iff for all $\Omega \in g$, if $\Gamma \mathfrak{R} \Omega$ then $M, \Omega \Vdash_v X$.

56.8) $M,\Gamma \Vdash_v \mathbf{P}X$ iff for some $\Omega \in g$, $\Gamma \mathfrak{R} \Omega$ and $M, \Omega \Vdash_v X$.

56.9) $M, \Gamma \Vdash_v (\forall x)X$ iff for every x-variant of v at Γ, $M,\Gamma \Vdash_w X$.

56.10) $M, \Gamma \Vdash (\exists x)X$ iff for some x-variant of v at Γ, $M, \Gamma \Vdash_w X$.

56.9) and 56.10) are crucial.56.9) makes sure that a relation hold for all members at a point for the universal statement to be true. 56.10) makes sure that a relation holds for some members at a point for the existential statement to be true. If we restrict ourselves to sentences (no free variables) then it is a theorem that if a sentence is true at a point Γ in a valuation then it is true at that point in any valuation.[174] We simply say that a sentence is true at Γ. As for the propositional case sentence is called *valid in a model*, if it is true at each point of the model and so on. We will now test some sentences for validity using this machinery.

[174] See (Fitting, Mendelsohn, 1998), pp. 98-99.

Scapegoat Example

Test 57)

$\Gamma \Vdash_v (\exists x)\mathbf{O}Q(x) \rightarrow \mathbf{O}(\exists x)Q(x)$

The following is a counter-model to the scapegoat example. Consider the augmented serial model consisting of Γ and Ω with $\Gamma\Re\Omega$ and $\Omega\Re\Omega$. $D(\Gamma)$=a and $D(\Omega)$=b.

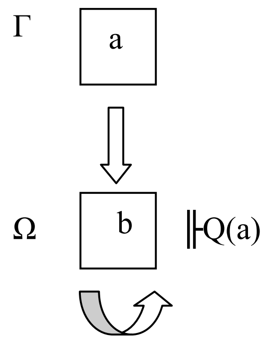

Figure 2. Counter model

Let *w* be the x-variant of *v* with w(x)=a. Now $\Gamma \Vdash_v (\exists x)\mathbf{O}Q(x)$ iff $\Gamma \Vdash_w \mathbf{O}Q(x)$. And the latter is true iff $\Omega \Vdash_w Q(x)$, ("only if" because a is the only object at Γ and w thus is the only x-variant there is). Since w(x) =a and a is in the relation Q at Ω, this is the case. On the other hand if $\Gamma \Vdash_v \mathbf{O}(\exists x)Q(x)$ then

$\Omega \Vdash_v (\exists x)Q(x)$.If this is true then there must be some x-variant of v with $\Omega \Vdash_w Q(x)$. Since there is just b at Ω we must have w(x)=b. This means that we must have Q(b) which we don´t. The contradiction proves *not*($\Gamma \Vdash_v \mathbf{O}(\exists x)Q(x)$) and therefore $\Gamma \Vdash_v \neg\mathbf{O}(\exists x)Q(x)$. Informally if Andrea(a) and Bert(b) are persons then Andrea exists in the actual world Reality (Γ) and Bert exists in the perfect world Utopia(Ω). Since Utopia is perfect nobody there is required to go to jail, because nobody there commits crimes. Now in the horrible actual world that Reality is, Andrea does not go to jail(we do not have $\Gamma \Vdash Q(a)$), because here money rules, legal systems are corrupt, she is a white, rich protestant and she has a devious lawyer.[175] However, in the perfect world Andrea *would be* required to go to jail, since she has committed a crime.(When the saint Bert discusses Andrea´s case with himself, he comes to the conclusion that if people like Andrea existed, they would have to go to jail. Then he goes to bed, happily assured that they don´t). As a consequence, in the actual world

[175] Naturally we would have to make the domain larger at Γ to formally include jails, lawyers and corrupt legal systems, but it is only the point to illustrate the example informally.

Reality it is true that it is obligatory for somebody to go to jail. But it not obligatory that somebody goes to jail, because in the perfect world Utopia where only Bert lives, nobody commits crimes and nobody goes to jail. I call it the scapegoat example, because Bert would be a scapegoat if the ethical or legal system required him to go to jail, even when he had not done anything.

Now a proof that the sentence $\Gamma \Vdash (\forall x)\mathbf{O}Q(x) \rightarrow (\exists x)\mathbf{P}Q(x)$ is valid in a serial frame for varying domain semantics. Assume $\Gamma \Vdash_v (\forall x)\mathbf{O}Q(x)$.Then for any x-variant of *v* we have $\Gamma \Vdash_w \mathbf{O}Q(x)$. Since the frame is serial we have $\Gamma \Vdash_w \mathbf{P}Q(x)$. Since w is *some* x-variant of *v* we have $\Gamma \Vdash_v (\exists x)\mathbf{P}Q(x)$, which concludes the proof. So if something is obligatory for everybody it is permitted for somebody.

Analytic Tableau Rules for Quantifiers

Again, I take the rules from (Fitting, Mendelsohn, 1998). To deal with existential quantification we introduce *parameters* or arbitrary names. If an existential sentence is true, there is something in the domain of the point, which is in the relation. That something we give an arbitrary name. If a universal sentence is true it will be true for that thing named too, whatever it is. We still only prove sentences. Since the domain may vary from point to point we introduce an infinite list of parameters, one for each prefix, s. For a prefix s we write parameters as a_s, b_s, … sticking to lower case letters from the beginning of the alphabet. Here are the rules.

$s\ (\forall x)\ \alpha(x)$	$s\ \neg(\exists x)\ \alpha(x)$
$s\ \alpha(a_s)$	$s\ \neg\alpha(a_s)$

Where a_s can be any parameter associated with the prefix s.

$s\ (\exists x)\ \alpha(x)$	$s\ \neg(\forall x)\ \alpha(x)$
$s\ \alpha(a_s)$	$s\ \neg\alpha(a_s)$

Where a_s is a parameter associated with the prefix s, which is *new* to this branch.

The rest of the rules are the same as for propositional SDL, except that the formulas that we attempt to prove are first order sentences. Here is a proof of the sentence $(\forall x)\mathbf{O}Q(x) \rightarrow (\forall x)\mathbf{P}Q(x)$.

58) $1\ \neg((\forall x)\mathbf{O}Q(x) \to (\forall x)\mathbf{P}Q(x))$
$1\ (\forall x)\mathbf{O}Q(x)$
$1\ \neg(\forall x)\mathbf{P}Q(x)$
$1\ \neg\mathbf{P}Q(a_1)$
$1\ \mathbf{O}Q(a_1)$
$1\ \neg\mathbf{O}Q(a_1)$
X

Santa Claus Example

Here is an attempt at a proof of the sentence $\mathbf{O}(\forall x)Q(x) \to (\exists x)\mathbf{P}Q(x)$.

59) $1\ \neg(\ \mathbf{O}(\forall x)Q(x) \to (\exists x)\mathbf{P}Q(x)\ )$
$1\ \mathbf{O}(\forall x)Q(x)$
$1\ \neg(\exists x)\mathbf{P}Q(x)$
$1\ \mathbf{P}(\forall x)Q(x)$
$1.1\ (\forall x)Q(x)$
$1.1\ Q(a_{1.1})$
$1\ \neg\ \mathbf{P}Q(a_1)$
$1.1\ \neg Q(a_1)$

Since the domains may vary this tree does not close. A formal counter model can be extracted from the failed tree. a exists at Γ, and b at Ω, as in 57) above. We have $\Omega \Vdash \neg Q(a)$ and $\Omega \Vdash (Q(b)$. Since b is in the relation Q everybody in the perfect world is. At the same time nobody in the actual world is permitted to Q since a is not in Q in the perfect world.

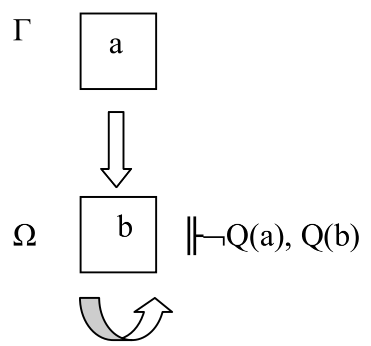

Figure 3. Counter model.

An informal account of the counter model in the same terms as above could go something like this. It is obligatory that all good people get a chance to be happy. Unfortunately there are no good people in the world Γ Reality, as we already know, because here lives only the wicked and thoroughly corrupted Andrea. So nobody exists at Γ, which is permitted to be happy. So even though it is obligatory that all be happy (everybody is happy in a perfect world, as Bert can agree, since Q(b) at Ω), it does not follow that there is somebody in Reality who is permitted to be happy. More serious and realistic examples could be made. It may be that it is obligatory that every excellent student gets a certain scholarship every year. Yet there may be no excellent students that year according to whatever standards the institution have. So nobody is permitted to get the scholarship. The example I first thought of is not in the serious category: Just because "It is obligatory that Santa Claus gives every good kid a present once a year" it does not follow that there is one single kid, who is permitted to get a present. They might all have been bad that year.

To conclude, varying domain semantics seems appropriate in a deontic context as Hintikka and others have argued.

A natural path to take now would be to go on to treat equality, functions, constants, singular terms, definite descriptions, the rest of the typical first-order logical machinery, but it is really not the purpose of this thesis to give an extensive treatment of first-order standard deontic logic, and besides it would in a sense be to cover new ground (and so fall out of the scope of a historical chapter), because many of these further topics have not really been thought through in a deontic setting, as far as I am aware. Instead I will now turn briefly to defeasible deontic logic.

Defeasible Deontic Logic

> The very expression "*prima facie* obligation" cries out for a defeasible or non-monotonic analysis. Such an obligation binds unless it is overriden by appropriate considerations – usually a higher obligation.[176]

Defeasible reasoning in an alethic setting deals with general belief, which may be defeated by more specific belief or knowledge. For normatives, we must distinguish between contrary-to-duty obligations, overriding obligations and violated obligations. The following example by Prakken and Sergot brings out the differences:

(1) There must be no fence.
(2) If there is a fence, it must be a white fence.
(3) is by the sea, there may be a fence.[177]

As pointed out by Prakken and Sergot, (3) overrides (1). It gives a licence. If there is a fence the Contrary-to-duty obligation, that it must be white incurs, by modus ponens. However, (1) is still violated, and the owner of the cottage may still get a fine, for instance. Contrary-to-duty or

[176] Nute, Donald, and Yu, Xiaochang. (1997). Introduction. In (Nute , 1997), p. 1.
[177] (Prakken and Sergot, 1997).(Nute, 1997) p.228.

secondary obligations do not override their primary obligations. On the other hand, (1) is the *prima facie* obligation and (3) is the overriding permission. In most cases (assuming most cottages are not by the sea) the prima facie obligation will be actual obligation, too. In case of a cottage by the sea, the overriding permission is "activated", overriding (1).(1) is "deactivated" it loses its normative force. It seems to be a principle of defeasible normative reasoning, that only normatives may override other normatives, here a permission overriding an obligation. There being a fence is not a normative but a descriptive, and though descriptives can violate or fulfil normatives they cannot override them. Prakken and Sergot describe defeasible arguments in the following way:

> An argument is called defeasible if, although valid on the basis of a certain set of premises, it might be invalidated if new premises are added.[178]

This description seems to cover the example with conditional overriding permissions as above. If a cottage is by the sea (extra premise), then the *prima facie* obligation not to have a fence is overridden. The description also points to the fact that defeasible logics are usually *non-monotonic.* A logic(such as classical propositional logic) is monotonic if, if a sentence is a consequence of a set, then the adding of new premises to the set does not affect this relation. On the other hand a now classical example from alethic defeasible logic shows this not to be the case here. "Birds fly" and "Tweety is a bird" implies "Tweety flies", but "Tweety is a bird and a penguin" does not. This could be because we have as rules "Penguins do not fly" ,"the set of penguins is a proper subset of birds" and a meta-principle of specificity that "more specific rules defeat more general rules when they lead to a contradiction" or something to this effect. In (Van der Torre, Tan, 1997) it is pointed out that there is a fundamental difference between deontic and alethic defeasibility.

> Although the obligation **O**P is violated by the fact ¬P, the obligation still has its force…[179]

A fact to the contrary cancels a general alethic rule, but a fact to the contrary only violates a general norm. This must be respected by a defeasible deontic logic.

These very general remarks do not do any justice to all the profound work done in this area. It is an exciting and interesting new trend in deontic logic, and I can only encourage myself and anybody else to study it further. In my opinion, however, there are aspects of deontic reasoning not especially linked to defeasibility. We have seen problems at a simple sentential level, the paradoxes, the semantic interpretation and so on.

[178] (Nute (ed.), 1997), p. 237.
[179] (Van der Torre, Tan, 1997). In (Nute (ed.), 1997), p. 79.

Evaluation of the Historical Survey

What did we get out of the historical survey undertaken in Part 1? In this section I present an evaluation. The first point is that normative sentences have to be treated in their own right. Many of the problems of normative logics arise because an attempt is made to fit normative sentences into a propositional theory of meaning or something derived from one. In such a theory true and false are the crucial concepts. As a consequence true and false are the only logical values of most systems. When reading the literature it becomes apparent that the two crucial semantic concepts regarding normative sentences are:

Force and Fulfilment

A normative sentence (or its semantic content)[180] may have *force* for somebody. This means that this person has to do what the norm prescribes, the prohibition prohibits. It will be difficult to reduce this “having to” to something else.

Permissions may be seen as a normative sentence without force or something with a different kind of force. Our intuitions about force for deontic sentences are reflected well in the square of opposition figure 1 above, if we replace obligations being true with obligations in force.

A normative sentence may also be *fulfilled* by somebody. This means that a person has done what a normative sentence prescribes. Semantically, it means that the corresponding proposition for an obligation is true. A normative sentence may also be violated by somebody. Our intuitions about fulfilment and violation can be seen in the following figure using the language of SDL.

$$\begin{tikzcd} \mathbf{O}Q & \mathbf{O}\neg Q \\ Q \arrow[u] \arrow[ur] & \neg Q \arrow[u] \arrow[ul] \end{tikzcd}$$

Figure 4. Fulfilment and violation table.

Here the diagonals are violation and the columns represent fulfilment in the direction of the arrows.

In the literature a “double-talk” results from these two facts:

1) The formal semantics of a normative logic is based on the concept truth.
2) The crucial semantic concepts for normatives are really force and fulfilment.

The logicians may intend to talk about force and fulfilment. Yet, they are forced by the formal semantics to talk about truth. Standard deontic logic is an obvious case in point. The following passage gives a bit of specific evidence for this claim. It is taken from an article written by leading experts. In the following, pay attention to the shift of language use, rather than the (good) points made by the authors.

> That is the reason why the deontic operator **O** had to be represented as a modal operator with a possible worlds semantics, to make sure that *both* the obligation and its violation could be true at the same time. Although the obligation **O**P is violated by the fact ¬p it still has its force, so to say.[181]

The explanation in terms of truth does not make any real sense and has to be clarified in terms of force, even though the correct (but informal) term *violation* is already used in the more formal account. It is my claim that the informal account is more than a simple explanation of the formal semantics as the case would be in an alethic setting where the informal and formal semantics correspond. The reason for this correspondence is that the formal semantics was developed for the alethic case. Let me present another example, where the situation is reversed. The authors start out talking the right way, but then notice how the shift into the formal account causes trouble.

> First, we want to determine the relative goodness of worlds by comparing which of the obligations in force they fulfil and which they violate. But what are the obligations "in force" whose violation or fulfilment is to be checked. As a first shot we will say that they are all the contextual obligations that are true in the model. [182]

The authors later change this "first shot" but that is not the point. The point is that there is no unproblematic way of going from the natural terms "violated", and "in force" to "truth". These are simply different semantic concepts, and it is not a mere matter of definition of "truth" to circumvent the trouble. Take proponents of standard deontic logic. They hang on to the idea of **O**P being *true* iff P is *true* in all perfect worlds. This talk blurs that what we are interested in, is **O**P being *in force* or *not in force*, and P *fulfilling* and ¬P *violating* **O**P. And these "informal" facts should be reflected in the formal semantics, because a formal semantics is a tool for regimenting our natural language ideas, not for putting them into some specific meaning theory that reduces every sentence of our language to something which is "true" or "false". This is to me one point of the *declarative fallacy* highlighted by Nuel Belnap, (Belnap, 1990). I diagnose the problems of most normative logics to be the problem of getting the formal semantics that we have to fit the informal semantics that we want. A way out of this, as I see it, is to develop a formal semantics which take the informal semantics

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[180] In Part II the distinction between a sentence and its content becomes crucial but this is an informal discussion.
[181] (Van der Torre, Tan, 1997), p. 79.
[182] (Prakken, Sergot, 1997), p. 252.

into account on a more basic and fundamental level than is usually done. This is what I do in the second part of this thesis.

PART II

A NEW SYSTEM

The word *system* in the title of part II of this thesis is to be taken in a broad sense, as a theory consisting of:

> …an…interdependent group of items forming a unified whole.[183]

In fact the system comprises two elements. A theory of meaning(A) and a logical calculus (B) based on the theory of meaning. The logical calculus is given two formulations. A semantic formulation justified by the theory of meaning, and a formulation via a proof procedure (semantic tableau). The proof procedure is a logical system in a more narrow sense of the term system. The basic rules of the proof procedure are sound and it is supposed complete with respect to the semantics. (See appendix 1).

A: A THEORY OF MEANING FOR IMPERATIVES

Since logic (taken in a strict sense) is concerned with inferences it may take semantic concepts such as truth for granted and concentrate on the forms of correct inferences.[184] In the same way I might take the semantic properties I am interested in concerning imperatives for granted, them being in force and fulfilled and violated, and simply start doing logic. Philosophically, this would not be a satisfactory approach, however. By a *theory of meaning for imperatives* I mean an investigation of semantic features of imperatives as they occur in ordinary language. In part this is carried out through a comparison with declarative sentences. This theory of meaning will serve as a philosophical justification of the significance these semantic concepts have in the logic of mandates(LM) developed in (B). LM is not strictly dependent on whether *this* theory is correct or not, but it does require some sort of justification (to be relevant philosophically).

Background: Propositions and Context-Free Declarative Sentences

A proposition is the content of a statement or declarative sentence. The ontological status of propositions is controversial. It may be taken to be a state of affairs or a meaning in the head or in

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[183] (Webster´s Dictionary, 1967), p. 895.
[184] Cf. (Dummett, 1991), p. 20.

some ideal realm. It may be the set of possible worlds where the sentence expressing it is true.[185] I do not need to commit myself to either or any of these suggestions. Here I will limit my interest to the *idealisations* made by most logicians to justify a logic of propositions or a *sentential logic.* I am getting at what Richard Jeffrey has called a *context-free sentence* and W.V.O. Quine called an *eternal sentence.*[186] These are free from all context-dependent variables, such as personal pronouns and temporal adverbs such as "tomorrow" or "now". Variables which may give the sentence different truth values in different contexts. Instead these variables have been replaced by constants, such as names of particular people and specific dates. In modal logic, some of these features are made variable again in a controlled form, but let us focus now on these eternal sentences. Apart from the removal of variables of the type mentioned, Jeffrey says:

> In logic it is harmless and useful to discuss context-free sentences in abstraction from their speakers.[187]

Such an idealised sentence expresses a *proposition* and is discussed by Frege:

> For Frege, a complete thought, a *Gedanke,* is an eternal, non-linguistic object whose truth value never changes.[188]

It is this idealisation or construction, which makes sure that the principle of bivalence holds for propositions. The principle of bivalence is subdivided into two further principles. These are, for a sentence X:

1) $\neg(X\wedge\neg X)$ *non-contradiction.*
2) $X\vee\neg X$ *tertium non datur.* [189]

In a bivalent logic such as classical sentential logic 1) and 2) conflate and are logically equivalent. The eternal sentences are supposed to justify the idea of two truth-values and that

> We shall attribute truth-values to context-free sentences…[190]

The important point here is not whether this is an adequate representation of what happens in ordinary language. The point is that this is a presupposition of classical propositional logic. We may argue that future contingents about a far future, for instance, cannot in any way be said to be determinately true or determinately false at the current point of time and that we cannot possibly put ourselves in this "godlike, eternal position", which Frege seems to propose, or equivalently we

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[185] Cf. (Taylor, 1998), p. 201: We can represent a proposition by *a set of possible worlds* – the set of possible worlds in which the proposition is true.
[186] (Fitting, Mendelsohn, 1998), p. 24.
[187] (Jeffrey, 1967), p. 5.
[188] (Fitting, Mendelsohn, 1998), p. 24.
[189] Cf. (Malinowski, 1993): p. 7.
[190] (Jeffrey, 1967), p.5.

might question the ontological status of essentially tense-less thoughts.[191] This is not a bad argument. Yet, we simply cannot express this philosophical conviction with the tools of propositional logic.. We need many-valued logic or modal, temporal logic. Propositional logic has a very limited *expressive power* compared to ordinary language but we still may want to keep it for purposes where the inevitable idealisations are harmless. I propose a pragmatic attitude to alternative logics. Appropriate idealisations may change in differing semantic contexts leading to different logics. This approach puts a lot of the justification into the "context" and the "idealisations" and these I will pay attention to in this chapter.

Imperatives and Propositions

As late as 1990 Nuel Belnap wrote:

> It is certainly my view, that our philosophical community is scandalously naive about imperatives, that our community wallows in non-socratic ignorance of imperatives.[192]

Fourteen years have elapsed since Belnap complained yet we still do not have a satisfactory linguistic theory of imperatives. In works on grammar or semantics and even pragmatics imperatives are often neglected. I suspect that a main reason for this is that they do not fit well into the standard paradigms, which often take statements as models. The alethic bias of the standard paradigm Belnap dubbed *the declarative fallacy.* [193]

As a basis of a logical system of imperatives we need an unprejudiced and workable semantic account of imperatives and in this chapter I supply one.

First of all, I will reject the thesis that imperatives are *propositions* with an extra imperative force added to them. Propositions in the philosophy surrounding classical sentential logic are expressed by context-free sentences and are either true or false. We cannot adopt this idea for imperatives. Imperatives are neither true nor false, so it seems confusing to claim, that their content is a proposition or that they have a propositional content. Furthermore, there is no obvious modification of this idea that yields a good semantic account for imperatives.

I will just give one example of why this solution is inappropriate. Consider the correspondence theory of truth, the idea that to every proposition corresponds some state of affairs. A very general way of presenting this for a propositional language L, is by means of Alfred Tarski´s T-schema all instances of which are meant to define what the predicate "is true" means in a language.[194]

3) T: s is a true sentence iff p

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[191] Prior had a similar idea at one point.

[192] (Belnap, 1990), p.17.

[193] (Belnap, 1990).

[194] Cf. (Taylor, 1998), p. 115: The aim is…to define truth predicates in systematic and rigorous ways…while preserving certain core elements of the correspondence theory of truth.

We get T-sentences by substituting names of sentences for s and the sentences themselves for p. For instance,

4) “Snow is white” is a true sentence iff snow is white.[195]

It may even be conceded that *any* theory of truth, correspondence or not, has to yield all the right instances of 3). [196]

We may try to use the T-schema above on the imperative sentence: “Run, Andrea!”.

5) “Run, Andrea!” is a true sentence iff Run, Andrea!

This obviously does not work. 5) is thoroughly ungrammatical[197] and does not make any sense. We may try to amend the schema through the concept of fulfilment of the imperative, while keeping the essential semantic concept “true”. Let “Andrea runs” be the corresponding proposition for the imperative “Run, Andrea!” as described in the section on Alf Ross above.[198]

6) “Run, Andrea!” is true iff Andrea runs.

Now we have a grammatical sentence but it is still bad. It is artificial to talk about either fulfilling an imperative or an imperative being in force as an imperative being true. Besides 6) would rule out most imperatives, which are issued before they are fulfilled. Usually Andrea is not going to be running when the imperative is issued(making it not true according to the definition), but the imperative may still be in force and it may be fulfilled later. Certainly an imperative has semantic value (is in force) when it is issued and not only after its been decided whether it is going to be fulfilled or not. If we try catching this dynamic aspect in the schema and write:

7) “Run, Andrea ” is *made* true iff Andrea runs.

Then we are left with a wholly new notion of *making* true which needs to be explained, and 7) is certainly not a T-schema, since this was meant to define true not making true.[199] This suggestion does not work but Hare is still right to say the following.

> An indicative sentence is used for telling someone that something is the case; an imperative is not - it is used for telling someone to make something the case.[200]

But “telling somebody to make something” and ”telling somebody that something is” are very different things and the former cannot be explained in the same semantic terms as the latter. Another problem is that this focus on the fulfilment of the imperative points in the direction of a logic of fulfilment with the problems this entails. (See historical chapter, Ross, Rescher).

This was meant to show that a common tool (T-schema) used for declarative sentences does not carry over well to explain the meaning of imperative sentences. One may argue that 5) being

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[195] (Taylor, 1998), p. 114.

[196] (Taylor, 1998), p. 114.

[197] Because implications and bi-implications in which both the antecedent and the consequent are imperatives are not grammatically well formed in English(or Danish or German). See also the section Imperatives in Force.

[198] Or more extensively in the next chapter.

[199] This point is due to Johanna Seibt.

[200] (Hare, 1952), p. 5.

ungrammatical is irrelevant. The fact remains that 5), 6) and 7) all make the assumption that an imperative can be a true sentence, which is plainly wrong. Take an imperative such as:
8)"Close the door, Andrea!"
I test some linguistic responses.
"Is 8) true Is it false? Is it possible? Is it necessary?" None of these alethic questions are appropriate. On the other hand, some responses are completely appropriate.
We might ask epistemic questions: Andrea might say: "Is it probable that I can do that given the circumstances?" This epistemic question regards the fulfilment of the imperative.
Furthermore she might ask the deontic questions: "Do I have to?"(Is it in force for Andrea or can an exception be made for her?). "Why do I have to?"(With what if any authority is this imperative given or how may it be rationally justified?). And responses like: "I am going to"(Andrea intends to fulfil the imperative) "I am not going to"(Andrea intends to violate this imperative) are also appropriate. All the appropriate responses considered so far regard the mandates being in force or being fulfilled or violated. Furthermore she might say things like: "I don´t have to because my father tells me no"(This imperative is defeated by an overriding authority.) These are claims about the imperative being overridden or defeated.

As Hare´s distinction between the phrastic and the neustic element shows statements and imperatives have a lot in common. Michael Dummett follows Hare and writes that statements and imperatives have a common content with different forces one called assertoric the other imperative.[201] This account explains that words like "four" and "plate" in sentences such as "There are four plates on the table" and "Put four plates on the table" have identical meaning.[202] Dummett does not think that either statements or imperatives are two-valued.

The linguistic responses that are appropriate for imperatives mostly had to do with their deontic features: "Do I have to do that?" " I am not going to do that" and so forth. So what is the relation between imperatives and deontic sentences?

Imperatives and Deontic Sentences

One argument may run as follows: Imperative is a grammatical category. It is the mood of a verb in a sentence and may be determined syntactically.[203] Deontic on the other hand is a semantic category. It has to do with how a semantic content (SDL takes it to be a proposition) is modified by the deontic modalities. Hence imperatives and deontic sentences differ because they belong to distinct linguistic categories. The argument is sound but it is not the intended outcome of a comparative analysis. Rather we mean to say something like: Given that there is a semantic content

[201] (Dummett, 1991),pp. 114-118. There is also an interrogative force.
[202] (Dummett, 1991), pp. 116.
[203] In (Hamblin, 1987) he points out that the imperative mood has to be determined in the context of a sentence, since there *is* no distinct imperative mood of English syntactically speaking. The infinitive form is used.

of a "context-free imperative" (in analogy to a proposition as the content of context-free declarative sentence) how would this semantic content relate to deontic sentences? Let us call this semantic content for a *mandate.* One question to ask is: What are the *context-free imperatives* that the mandates are supposed to be the contents of?

Before I try to answer this, I briefly present a few theories about how the relationship between imperatives and deontic sentences have been envisaged in recent philosophy.

Carnap and other positivists argued that deontic sentences are really concealed imperatives.[204] Hare argued that deontic sentences imply imperatives.[205] Some claim that the difference between an imperative and a deontic sentence is that the latter can be true. In my view it is a mistake. It only confuses reason to bring the semantic concept of truth into normative discussions, as I believe we have seen in connection with the formal semantics for SDL. The evaluation rules make a lot more sense when we think in terms of obligations being in force and being fulfilled and being violated. I also believe that the linguistic responses to imperatives shown above suggest that these should be thought in connection with deontic sentences rather than apart from them.

A more recent theory by M.A.K. Halliday links deontic sentences and imperatives intimately.

> Halliday (1985) says that epistemic modality calibrates the area of meaning between *Yes* and *No;* whereas deontic modality calibrates the area of meaning between *Do it!* And *Don´t do it!*[206]

I do not know whether this is true for epistemic modalities (no pun intended). However, for normatives, this hypothesis suggests that if we disregard the area of meaning "between" the Do and the Don´t we get these as duals and bivalence for semantic values connected to them. The Do´s and Don´ts are imperatives. The "area of meaning" between the Do! and the Don´t! is expressed by the deontic sentences.[207] This view leads in the direction of letting imperatives play the role of propositions or context-free sentences and letting deontic sentences play the role of modally qualified propositions in modal logic or as when a proposition is given a degree of probability. Deontic sentences put a degree on the imperative force. If you must do something it is stronger than if you ought to do it, which is stronger than that you may do it. If I am right in taking the theory this way, we still need an account of what it is for an imperative to be context-free and to see whether this yields bivalence. Let us first look at some situations where it might be useful to assume bivalence giving a bit of pragmatic motivation before the theoretical explication. When trying to

[204] (Hamblin, 1987), p. 18.
[205] (Hare, 1952)
[206] (Cruse, 2000), p. 287.
[207] (Halliday, 1985), p. 335.

make a decision an imperative in form of a Do or a Don´t is often appropriate as a guideline. I consider an agent perspective and an authority perspective.

Decisions Made by an Agent and for an Addressee

Decision theory is a huge, independent area of research. Here I will only focus on one agent situations with disregard to probable utility of choices, distinctions between risk and uncertainty and so on.(See e.g. (Horty, 2001) for decision theory in a deontic setting). I am only trying to link decisions to imperatives from the point of view of an agent, with a certain set of propositions given as circumstances and a certain set of imperatives given as normative guidance. I am arguing that in situations of this kind you often need a Do or Don´t answer with no middle ground. The most obvious middle ground is "you may do this" a permission, which in this context boils down to "you decide!". There are situations where this is entirely appropriate. If for instance, you are trying to see whether a certain set of rules demands some action of you or not, the not demanding will be a permission not do this action. On the other hand, there are situations where you need to act and where not-acting is also acting. Here you need a straight answer. And if we want a straight answer of this sort we need bivalence of imperatives and Do and Don´t as duals.

Consider the following two inferences.

9)	10)
Kiss him only if you love him!	You have to kiss him only if you love him.
You do not love him	You do not love him.
______________________	__________________________________
Don´t kiss him!	You do not have to kiss him.

In both inferences the conclusion is given from the premises by modus tollens. But in 10) the conclusion is a permission not to kiss him, whereas in 9) the conclusion is an imperative. 10) is valid in SDL. But in order to get the conclusion of 9) we need bivalence for imperatives or in other words we need Do and Don´t to be contradictories. Not just contraries as "You have to kiss him" and "You have to not kiss him" are in SDL. Assuming that there are situations where we want to reason as in 9), and I believe there are such situations, here is some pragmatic motivation for considering an imperative logic with bivalent imperative force values.

The other situation I can think of is when a norm giver wants to take any relevant decision for the addressees of the imperatives. This is not to say that any action of an addressee has to be considered normatively which would be absurd and mean total control and we do not even have this over our own actions. It is only to say that *if* an action (or a condition) is considered normatively then a straight answer is given to the addressee in form of a Do! or a Don´t! No room is left for choice in the normative sphere. Now I turn to what I regard the most important result of the historical investigation in the previous chapter, which is that the most important semantic features of imperatives is that they are in force or not in force and fulfilled or violated.

Context-free Imperatives

A context-free imperative is one where all possible variables with regard to addressee, time of force, time of possible fulfilment, etc. have been replaced by constants. In particular I will assume that the following have been determined.

11) *Addressee*. It is determined which object the imperative applies to. In ordinary language this is often done by adding an apposition as in: "Run, Andrea!". In the logic of mandates (LM) presented next chapter this object is often called "you", which counts as an arbitrary name for some particular person or well-defined group, which the particular imperative applies to.

12) *Window of possible fulfilment*. It has to be specified, *when* an imperative can be fulfilled. A window of possible fulfilment has a starting point and an end point and within this window the addressee has to fulfil the imperative. All temporal variables are assumed removed in this specification. If for instance the window of possible fulfilment is "between 1 and 3 on a certain date" and the imperative is "Go for a run, Andrea" Then Andrea has to go for a run between 1 and 3 that day. After the window of possible fulfilment we regard the imperative as settled with regard to fulfilment. This means that it is either violated or fulfilled at this point, and bivalence holds between violation and fulfilment. In LM we will model imperatives at this point *after* the window of possible fulfilment.

13) *Authorization.* An imperative is either authorized or not. What this means is that some authority backs it up or not. The authority corresponds to a *fact* or a *state of affairs* in the philosophy of classical propositional logic. It gives an imperative its imperative force value, as a fact gives a statement its truth value. What an authority is I will not attempt to define, only that it is to be understood in a very broad sense. The only assumption I wish to make is that there are authorities and that they authorise imperatives. It may be a legal institution or an official person, but it may also be conscience, reason, morality, tradition or even inanimate objects such as a book (e.g. the Bible). The authority can be an authority in its own right (a robber with a gun) or because it is endorsed by the addressee (e.g. deciding to take advice from somebody) but this distinction is not clear cut. Its imperatives may be rational and justified or unfair judged by some evaluative meta-

standard of rationality, justification and so on.[208]Although I take authorities as primitives for the purpose of this thesis they are not in principle irreducible.[209] The analysis of authorities is taken further in sciences such as history, sociology, psychology, legal science and so on and in other areas of philosophy. In a theory of meaning they represent a natural place to stop. In LM we *only* model imperatives which are authorized. Furthermore we make the important idealisation that an authorization consists in either a prescription or a prohibition. If an imperative is modelled it is authorized. And if an authority authorizes an imperative in LM it does it with a Do this or with a Don´t do this not with a "you decide". We have looked at a few situations above where this idealisation seems appropriate.

When all these variables have been decided we have an imperative which is in force for somebody backed up by an authority and which needs to be fulfilled within a well-defined temporal window. It seems fair to call this a context-free imperative. We shall say that a context-free imperative expresses a *mandate*, that a mandate is the content of a context-free imperative. We give mandates one and only one force of two values, a negative imperative value or a positive imperative value. The negative value corresponds to a Don´t. The positive value corresponds to a Do. This is important because it now we have *tertium non datur* for the mandates considered. In the next chapter I develop a formal logic of mandates.

Sentential Connectives

> …logicians have only ever thought "truth-functional" was a good descriptive term because they were wearing indicative blinkers.[210]

In ordinary language there are pure and mixed sentences involving mandates and the sentential connectives. There are mixed sentences such as "If you go to the beach, put sun-screen on!" where the antecedent is a proposition and the consequent a mandate. I take it that mixed sentences are forms of mandates and they are considered so syntactically and semantically in LM. The meaning of words like "and", "or" and "if..then…" do not seem to change much when dealing with different types of sentences in ordinary language. In most logics (but not all, e.g. not in intuitionistic logic) sentential connectives are defined almost the same way. Going from propositional to predicate to alethic modal logic, for example the meanings stay very similar, even though logical concepts such

[208] With regard to justification it seems like imperatives either are judged to have intrinsic value or value in regard to the consequences of their fulfilment. A justified law may be one that avoids bad consequences for society and produces good ones, or it may be a law that expresses a certain intrinsic value for instance of human rights, even if the consequences of fulfilment cannot be foreseen.

[209] The *states of affairs* or *facts* of classical logic are analysed further by sciences such as physics, biology and so on.

[210] (Hamblin, 1987), p. 72.

as validity are changed. In LM I exploit this intuition and let the sentential connectives be exactly similar functions for mandates and propositions. Sentential connectives are functions from truth values and imperative force values to truth values and imperative force values. Informally, this expresses the philosophical conviction that our reasoning with the connectives is identical when dealing with mandates and with propositions. Naturally the term "truth-functional" will have to be abandoned as too narrow. It is not so important to coin a new phrase, but perhaps "logical value functional" will do. One purpose of classical propositional logic is to find the truth value of complex sentences from the truth value of simpler sentences. One purpose of LM is to find the logical value of complex sentences from the logical value of more simple sentences be they mandates or propositions.

Imperatives in Force

First a remark about the semantic category of (normative) force, which is represented with two imperative force values corresponding to Do and Don´t in LM. Force value is a semantic category and not a pragmatic category as the illocutionary force of speech-act theory. This is why we can do logic with it. It is not connected to the meaning of specific performances or utterances of sentences but to meaning regarded independent of context of utterance. The meaning of a sentence does *not* determine its illucutionary force.[211] In a context it may depend on intonation (showing whether I speak literally or not, for instance), certain non-verbal gestures and so on, whether I promise or direct or offer or assert or perform some other illocutionary act. But the meaning of a context-free imperative is meant to determine its imperative force value independently of such factors, however interesting they are.

Consider what I will call atomic mandates. *Atomic* mandates can be expressed by imperative sentences without sentential connectives: "Catch the ball!", "Know yourself!", "Go to school!", and so on. We take the actual sentences to be abbreviations of context-free imperatives. It seems now for a certain object y and any member X of a set of atomic mandates that all instances of the following Imperative Schema or *I-schema* holds. I write Xy for X applies to y.

14) "Xy" is in force iff y has to do the following: X.[212]

A specific instantiation would be:

"Run, Andrea!" is in force iff Andrea has to do the following: Run!

[211] Cf. (Bach, Harnish, 1979), p.10: …the slogan "Meaning determines force" is generally false.

[212] The I-Schema is related to Kant´s idea that imperatives expresses an ought and it is also loosely related to Hare´s idea that deontic sentences implies imperatives.

I have thus reduced force to a question of "having to do".[213] The point of the schema is that the force of imperatives can only be clarified by some deontic quality. I could also have used another deontic term such as obligatory.

15) "Xy" is in force iff it is obligatory for y to do the following: X

"Andrea, Run!" is in force iff it is obligatory for Andrea to do the following: Run!

I do not think the force can be reduced any further at least not to sentences with a truth value and this goes for imperatives and for deontic sentences, so there is my stand on the relation between the two. A bad attempt is to reduce imperatives or deontic sentences to "descriptions of norms". Either these descriptions must be declarative sentences describing non-linguistic obligations in which case somebody will have to explain which fact corresponds to these.[214] Otherwise they are descriptions of obligations as *expressed by some means*, e.g. language. In the latter case their truth value will depend on whether the *description* is adequate or not. Something may be false description of a deontic sentence, without the norm being not in force or violated. If we try to limit ourselves to "true descriptions" the truth-value is fixed and we cannot do normative logic with just one value. In normative logic we are interested in what these descriptions describe, normatives in force or not in force, and we have to go to this semantic level, staying on the meta-level of descriptions will only cause mistakes. A similar attempt is a reduction to "given orders". It is thought that these can be verified and thus given a truth-value. It is very important to see the fundamental circularity in this approach. Say that you want to see if a law has normative force by checking whether it is a given law. Certainly, this is an appropriate thing to do to verify whether a mandate is in force for you or anybody else or as Alf Ross called it "legitimise" a certain mandate. But it will not do as a philosophical justification or definition of what normative force is. You check for a certain imperative or deontic sentence amongst a set of imperatives or deontic sentences. But then you still need to explain why the set of imperatives and propositions have normative force, because including a normative sentence in a set of other normative sentences does not in itself give it force. It seems like they have to be not just given in a factual sense, but that they have to be given by an authority, which backs it up. And then we are back to the language of force and fulfilment and something like the I-schema or the explanation of authorisation above. This reduction must fail and it does not make any sense to give a truth value to "given" mandates, because them being "given" is

[213] We can get cases like "Andrea has to do the following: be tall!" But the strangeness probably comes from the strangeness of the imperative "be tall!" rather than the schema.

[214] In this case the T- schema as in: "you have to run" is true iff you have to run, *which presupposes* that deontic sentences can be true is adequate, but it is this presupposition I question: What are the normative facts?

not a sufficient condition for their normative force. Unless we operating with a normative concept of "given" meaning: "issued and backed up by an authority" in which case we are again back to something like the I-schema.

Imperatives Fulfilled

The operator analysis for imperatives !Q makes it easy to represent fulfilment. When !Q is settled(after the window of possible fulfilment) !Q is fulfilled when Q is the case and violated when $\neg$Q is the case. I have however rejected the prevalent semantic analysis behind the (syntactic) operator analysis that an imperative is a proposition with an added force. I do agree with Dummett and Hare that imperatives share a common semantic content with the proposition fulfilling them. I do not need the operator analysis since in LM the mandate Q and its negation $\neg$Q are taken to be contradictories and no two kinds of negation are distinguished (leaving out permissions)[215]. I will use a two-sorted language with uppercase letters for mandates and lower-case letters for propositions. Furthermore I define the relation between a mandate and its corresponding proposition or action (fulfilling the mandate) rather than presupposing it, as it is tacitly done with the operator analysis. This way for a mandate Q the proposition q will fulfil it and $\neg$q will violate it. I have however not taken for granted that there is exact semantic correspondence between a mandate and a proposition fulfilling it, as a matter of fact their semantic contents may differ. However, in the intended interpretation a mandate and its corresponding proposition will share a common content. This content is no more a proposition than a mandate, however it is at a more basic semantic level than what makes a sentence a mandate or a proposition. It is important to see that I do not need to present or commit myself to a semantic theory for this to justify the logical theory because I define the relation in the formal system rather than presupposing it. It seems like SDL and other systems using the operator analysis *do need* to supply such a semantic theory, which is actually very rarely done – if ever.

With the theory of meaning presented I will now go to the formal system: The logic of mandates, which is a sentential logic of propositions and mandates.

B: A LOGIC OF MANDATES (LM)

Mandates and Imperative Force Value

A *mandate* is an imperative version of a proposition. Mandates are idealised imperatives. A mandate has one or several *addressees*, typically a set of people, which it applies to. This may be unspecified in a grammatically imperative sentence. In this chapter I develop a simple *logic of mandates* (LM). In classical propositional logic one deals with truth functional consequence. In LM we keep classical propositional logic as a sub-logic, but we also want to treat mandates. The semantic concept we are interested in with regard to imperatives is not truth. Rather imperatives cannot be true or false. As a consequence truth is not going to be a semantic concept of our formal semantics for mandates either. Rather, we are interested in mandates in force, which are either fulfilled or violated. In LM we assume one and the same addressee throughout an inference. This addressee is usually informally called "you" in the following but this "you" is to be taken as an arbitrary name for some particular person or object that the mandate applies to. Much more controversially every mandate has an *imperative force value*. Informally this means that the imperative is in force for the person it applies to. Force value either has positive or negative polarity. What this means is that a principle of bivalence holds for mandates. The principle of bivalence can be subdivided into two principles which both hold for a mandate X:

1) $\neg(X\wedge\neg X)$ *non-contradiction.*

2) $X\vee\neg X$ *tertium non datur.*

2) is more controversial than 1). It may be felt, and perhaps rightly, that there *is* a third option, which is that X is not in force at all, something corresponding to being permitted not to X. Mandates differ from the obligations and permissions of standard deontic logic in this way. Because, whereas 1) holds for obligations and 2) for permissions, 2) does not hold for obligations, and 1) not for permissions. We give mandates one of two force values. Classical propositional logic only deals with propositions, which are true or false, and not for instance with future contingents, taken as sentences without a definite truth-value. In LM inferences like 3) and 4) turn out valid.

3) Kiss him only if you love him!	4) Have a banana if you feel like it!
You do not love him.	You feel like it.
______________________	______________________
Do not kiss him!	Have a banana!

[215] The two kinds of negation is the best reason for the operator analysis but these can be represented without the assumption that the operator operates on a proposition as it is done in MDL for instance where it operates on actions.

The logic of mandates presented will not be able to distinguish between a permission and an order. Some may feel that 3) and 4) are not formalised right in the following. In standard deontic logic we would have to formalise “Kiss him” in the first premise as **P**K to get the conclusion ¬**P**K, which seems the right formalisation of the conclusion. On the other hand, if we formalised “Kiss him” as **O**K, we could only conclude ¬**O**K, which would be interpreted “You do not have to kiss him”. Here, in the logic of mandates, we always get a “straight” answer. “Do it!” or “Don´t do it”. LM might be said to be perfect for people who do not want to think for themselves. Some may see 3) and 4) as misleading or peculiar ordinary language sentences, as Hamblin did, when he called sentences like 4) permissives expressed by imperatives. They would probably formalise them along the lines suggested above for SDL. Others may simply regard 3) or 4) or both as invalid.

Mandates and Fulfilment Value

I have talked a bit about mandates and mandates in force. The other crucial feature of mandates is that they can be carried out, fulfilled, obeyed, etc. In LM a mandate has one of two fulfilment values, *fulfilled* or *violated*. Again a principle of bivalence will hold. Again, it may be felt there is a third option, or that a mandate, for instance may be unfulfilled without being violated. This idealisation is a bit easier to justify than for the force values. Any mandate must have a *window of possible fulfilment.* This might be thought of as the time when it is possible for the addressee to do what she is told.[216] It is part of our idealisation that any mandate has such a specified window. It may be that you are under order to “Go for a walk between two and three in the afternoon!”on a certain day”. After the window of possible fulfilment we say that the mandate is *settled* with regard to fulfilment. It is at this point we choose to model the mandates in our system. For the example above, this happens at three o´clock that certain day. After three it is no longer possible to fulfil the mandate so if it is not already fulfilled it is violated. On the other hand, if it is fulfilled at this point it is not suddenly going to be violated, or so we assume. So the principle of bivalence holds. Fulfilment and violation are not features decided by a mandate alone. It is a binary relation between a mandate and its *corresponding proposition.* It is another idealisation that we are going to assume that there is such a corresponding proposition for each mandate, a proposition describing what the mandate is prescribing. We also speak of corresponding propositions as settled, meaning that they are true or false at the end of the window of possible fulfilment. When it is true (and settled) the

[216] There may be some dynamic relation between different “normative states” with different windows of fulfilment. As an example take it that you have to do something “before you press the button”, after doing which you have to do something else. We could call this two different normative states separated by the action of pressing the button.

mandate is fulfilled. When it is false (and settled) the mandate is violated. Any mandate has a corresponding proposition, but some propositions do not have "corresponding mandates". We may want to regard some things impossible to order of somebody, e.g. 2+2=4. To sum up, we are making certain idealisations to create an entity corresponding to a proposition of classical propositional logic. We attempt to create an "eternal context-free sentence".

5) The first idealisation that we always take the mandate to have a specified non-empty set of addressees that the mandate has as its domain. Furthermore we are taking it to be specified exactly, which object or objects from this set a given mandate applies to. For the inferences that follow I take the addressee to be the same throughout an inference. This is not essential but makes a sentential language easier to make. Informally we often call this object "you". This idealisation has the consequence that it is not dependent on a context outside of the mandate to whom the mandate applies.
6) The second idealisation is that we only consider mandates in force. The purpose of this is to ensure the principle of bivalence for mandates. Of the two parts of the principle of bivalence *tertium non datur* is more problematic. What this means is that for any mandate, the person either has to fulfil it or has to fulfil its negation. Behind this lies an intuition that permissions can be taken as mandates *not* in force. We do not deal with permissions in the simple logic that LM is.

7) The third idealisation is that we model the mandates at a point where they are settled. When a mandate is settled it is either fulfilled or violated. To make sure that a mandate is always settled it is assumed that any mandate has a window of possible fulfilment. After a mandate cannot any longer be fulfilled we take it to be violated. If it is fulfilled at this point we also take it that it cannot be violated. Fulfilment is also bivalent and is a relation between a mandate and its corresponding proposition, the proposition describing the state of affairs or event or action, which the mandate is prescribing.

We may expect inferences like 3) and 4) to come out valid. Furthermore we expect LM to be able to treat the deontic paradoxes. Furthermore we expect LM to show when a mandate is violated and when it is fulfilled in a valuation. This in turn leads us to say something about moral dilemmas, which we take to be a set of mandates and propositions that are such that any valuation must violate at least one of them. A set of mandates that simply cannot be fulfilled on logical grounds is

contradictory. But the world (the propositions) may be such that one of a set of mandates is violated. Some of these situations are called moral dilemmas.
I think it is time to go to the formalities.

Language of LM

We need the usual sentential connectives. $\neg$ is a unary connective. $\wedge$, $\vee$, $\rightarrow$,$\leftrightarrow$ are binary connectives. We also need to parentheses),(. Then we need two separate lists of variables making this a two-sorted sentential language. The terms of one list is to be interpreted as mandates with force and fulfilment values, the others as propositions with a truth value. The first list is formally taken to be Q, Q_0,Q_1, ... called imperative variables. Informally I use any upper-case letter to represent mandates. The other is taken to be q,q_0,q_1, … and they are called propositional variables. Informally I use any lower-case letter to represent propositions.

The formulas of the language of LM is the least set obeying the following conditions, called *formation rules*:

8.1) Any propositional variable is an (atomic) propositional formula.
8.2) Any imperative variable is an (atomic) imperative formula.
8.3) If X is a propositional formula, then $\neg$X is a propositional formula
8.4) If X is an imperative formula, then $\neg$X is an imperative formula.
8.5) If X and Y are propositional formulas and *b* is a binary connective then (X*b*Y) is a propositional formula.
8.6) If X and Y are formulas and X or Y is an imperative formula and *b* is a binary connective then (X*b*Y) is an imperative formula.[217]

As in classical logic we may omit outer parentheses.
The set of formulas are divided into imperative formulas and propositional formulas. The above conditions ensure that if any formula contains an imperative formula it is itself an imperative formula. Otherwise, it is a propositional formula.
The sentences we wish to model are of two kinds. One is called mixed and will be sentences like the first premises of 3) and 4) above. The others I call pure and they can either be propositions like the second premises of 3) and 4) or pure mandates like
"Kiss him or leave him!". We face the problem that there are no mixed conditional or biconditional mandates in ordinary language. Sentences like

[217] This also makes (X*b*Y) an imperative formula if X and Y are both imperative formulas.

9) “If Run, then put on your shoes!” are not well formed.

On the other hand $(R \rightarrow S)$ is well formed in the formal language, and we may interpret them as we would $(\neg R \vee S)$. If $(R \rightarrow S)$ symbolised what we attempted to say in the sentence 9) above, we would interpret back into ordinary language as “Don´t run or put on your shoes”.

Corresponding Proposition Defined

As mentioned above we assume that each mandate has a corresponding proposition. Let the mandate X be “Run before nightfall!” and say it applies to you in the window of fulfilment indicated between now (whenever that is) and nightfall. Then if “you ran before nightfall” is true right after nightfall, then the mandate is settled and fulfilled. If “you ran before nightfall” is false at that time, the mandate is settled and violated. “You ran before nightfall” is called the corresponding proposition. We define a syntactic relation between imperative formulas and their corresponding propositional formula as follows.

10) For any atomic imperative formula X on the place n in the list of imperative variables, the propositional variable Y on the place (2n-1) in the list of propositional variables, Y is the corresponding propositional formula for X.

(So q_0 is the corresponding proposition for Q_0, q_2 is the corresponding proposition for Q_1, and in general q_{2n} is the corresponding proposition for Q_n.

11) If X is an imperative formula, then the result of replacing all imperative variables in it with their corresponding propositional formulas is the corresponding propositional formula of X.

10) and 11) ensure that each imperative formula get one and only one propositional formula assigned to it, and so together they describe a function from the set of imperative formulas to the set of propositional formulas. But since every other propositional variable is not the corresponding proposition there are some propositions, which are not corresponding propositions. Informally I write a mandate and the corresponding proposition as an upper-case letter and the corresponding lower-case letter from the alphabet. So I write $R \wedge r$ for the mandate R and its corresponding proposition, for example. When a propositional letter is written alone in an inference we take it that there is no corresponding mandate for it.

Different imperative formulas get the same propositional formula assigned to them. In particular $(Q_0 \wedge Q_3)$ and $(q_0 \wedge Q_3)$ will both get $(q_0 \wedge q_6)$ assigned to them. What this will mean in the semantics is that for instance the pure mandate “Run and eat!” and the mixed “You run, but eat too!” are both fulfilled by the proposition “You run and eat”.

Formal Semantics of LM

Time has now come to lay out a formal semantics. For propositions we simply want to take over classical semantics. But for mandates we want something else. We want mandates to have either negative imperative force value or positive imperative force value. This may informally be thought of as connecting a “do” or a “don´t” with each mandate, since I have ruled out any third option. If a “do” is connected with a mandate X it has positive force and it has to be fulfilled. If a “don´t” is connected with a mandate X it has negative force and its negation $\neg X$ has to be fulfilled.
We want to operate with six semantic values. True and False for propositions represented 1 and 0 and called *truth values*. A positive imperative force value represented i. A negative imperative force value represented ø. Together they are called *imperative force values*. A fulfilment value represented f and a violation value represented v, called practical values. At first we concentrate on the four first mentioned values, called logical values, since those are the ones involved in traditional logical relations in LM. The latter two we call practical values, as mentioned above.

Valuation

A valuation is a function from a set M of formulas which assigns one logical value to each formula of the set. We make it a requirement of the set M and an imperative variable X and its corresponding propositional variable x that if M contains an atomic imperative variable X, then M also contains corresponding atomic propositional variable x. Logical values are assigned to all formulas of M in the following way.
12.1) Each atomic propositional formula has one truth value assigned to it.
12.2) Each atomic imperative formula has one imperative force value assigned to it.
12.1) and 12.2) together is called an *interpretation* of a given set of atomic formulas.
We are only interested in the valuations obeying the following further conditions called valuation rules, which extend an interpretation to all formulas of LM. 1 and i are called *designated values.* 0 and ø are called *anti-designated values*. For formulas X and Y imperative formulas are only assigned imperative force values and propositional formulas are only assigned truth values.
Furthermore,
12.3) $\neg X$ is assigned designated value iff X is assigned anti-designated value.

12.4) $(X \wedge Y)$ is assigned designated value iff X is assigned designated value and Y is assigned designated value.

12.5) $(X \vee Y)$ is assigned designated value iff X is assigned designated value or Y is assigned designated value.

12.6) $(X \rightarrow Y)$ is assigned designated value iff if X is assigned designated value then Y is assigned designated value.

12.7) $(X \leftrightarrow Y)$ is assigned designated value iff $(X \rightarrow Y)$ is assigned designated value and $(Y \rightarrow X)$ is assigned designated value.

We can use the valuation rules to find the value of a given formula in a valuation. Say the valuation assigns i to Q and 0 to r. According to the formation rules $(Q \rightarrow r)$, $(r \rightarrow Q)$, and $(Q \vee r)$ are all imperative formulas. So they must have imperative force values assigned to them. Since Q has designated r must have designated value for $(Q \rightarrow r)$ to have designated value. Since r has anti-designated this is not the case and so $(Q \rightarrow r)$ has anti-designated value, ø. In the same way it can be checked that $(r \rightarrow Q)$, and $(Q \vee r)$ both get designated value i.

Validity in LM

If a formula gets designated value in all valuations we say that it is a valid formula of LM or a tautology. If it gets anti-designated value in all valuations we call it a contradiction. If it gets designated value in one valuation it is called satisfiable.

Since each atomic formula gets one of two values then for a finite set of n atomic formulas, there will only be 2^n possible interpretations, which can be extended to one and only one valuation according to the valuation rules. It is a finite matter to check whether any formula is valid or not. We say that LM is *decidable* with regard to validity.

Consequence in LM

For a set of formulas S and a formula X if there is no valuation giving designated values to all members of S and anti-designated value to X, we say that X is a consequence of X. We also say that S entails X or that we infer X from S. It is also decidable whether a formula is a consequence of a set of formulas or not.

Fulfilment and Violation of Mandates in LM

As mentioned we are dealing with mandates, which are settled and so either fulfilled or violated. In the construction of the valuations above we have already made sure that the corresponding atomic propositional formula for an atomic imperative formula is given a truth value. It follows that for any imperative formula its corresponding propositional formula is given a truth value in a valuation, because if a formula is given a truth value its negation is, and for two formulas if they are given values the binary formulas made from them are, and one of these will be the corresponding propositional formula for the imperative formula. We now use this fact to make valuation rules for fulfilment and violation in a given valuation.

13.1) For any imperative formula X and its corresponding propositional formula x, if they are both given designated or both given anti-designated value the imperative formula is also given the value f in that valuation. We say that X is fulfilled in the valuation.

13.2) For any imperative formula X and its corresponding propositional formula x, if one formula is given anti-designated value and the other designated value, we also give the imperative formula the value v. We say that X is violated in the valuation.

Note that a violation is *not* the same as a contradiction. It is perfectly consistent for mandates to be violated, it happens constantly in ordinary life.

Logical Value Tables for LM

The following logical value tables can be helpful when testing formulas for validity or sets of formulas for consequence.

	$\rightarrow$				$\vee$				$\wedge$				$\leftrightarrow$			
A\B	**1**	**0**	**i**	**ø**	**1**	**0**	**i**	**ø**	**1**	**0**	**i**	**ø**	**1**	**0**	**i**	**ø**
1	1	0	i	ø	1	1	i	i	1	0	i	ø	1	0	i	ø
0	1	1	i	i	1	0	i	ø	0	0	ø	ø	0	1	ø	i
i	i	ø	i	ø	i	i	i	i	i	ø	i	ø	i	ø	i	ø
ø	i	i	i	i	i	ø	i	ø	ø	ø	ø	ø	ø	i	ø	i

	$\neg$
A	
1	0
0	1
i	ø
ø	i

The following is an example of how the tables may be used. Consider the inference:

17) If we need milk, run to the store!
Don´t run to the store!
____
We don´t need milk

Formalising as follows,

q: we need milk

R: Run to the store!

we can formalise the premises and conclusion as in the following table.

17.1)

R	q	$q\rightarrow R$	$\neg R$	$\neg q$
i	1	i	ø	0
i	0	i	ø	1
ø	1	ø	i	0
ø	0	i	i	1

Here the four bottom rows in the schema are the four possible valuations. In each of them, when the premises ($q\rightarrow R$) and $\neg R$ are given designated value, which is only the case in the fourth row so is $\neg q$, and so according to the definition of consequence the premises entail $\neg q$. [218]

Analytic Tableau Rules for LM

The main reasons for using a system of analytic tableau is that it simplifies testing formulas and inferences for validity, while maintaining the mechanical ease of logical value tables.
Analytic tableau rules are based on the definitions laid down in the evaluation rules, and the system they define can be proven to be consistent and complete with regard to our definition of validity above. The following are the basic rules of the system. α and β are formulas.

Basic Rules

DN-rule	$\wedge$-rule	$\vee$-rule	$\rightarrow$-rule	$\leftrightarrow$-rule
$\neg\neg\alpha$	$\alpha\wedge\beta$	$\alpha\vee\beta$	$\alpha\rightarrow\beta$	$\alpha\leftrightarrow\beta$
α	α	$\alpha \quad \beta$	$\neg\alpha \quad \beta$	$\alpha \quad \neg\alpha$
	β			$\beta \quad \neg\beta$

[218] This particular inference has been discussed in the literature. It goes against *Hare´s principle* that no proposition can be inferred from imperative premises, which cannot be inferred from those of the premises which are propositions. Rescher(1966), p. 96 allows the inference as an *antilogistic inference*.

¬∧-rule	¬∨-rule	¬→-rule	¬↔-rule
$\neg(\alpha\wedge\beta)$	$\neg(\alpha\vee\beta)$	$\neg(\alpha\rightarrow\beta)$	$\neg(\alpha\leftrightarrow\beta)$
╱ ╲	$\neg\alpha$	α	╱ ╲
$\neg\alpha$ $\neg\beta$	$\neg\beta$	$\neg\beta$	α $\neg\alpha$
			$\neg\beta$ β

When the rules are used logical value trees unfold. When a rule is used on a formula that formula is *checked.*

A branch in a tree is *closed* when a formula and its negation occur in that branch. A tree is *completed* when all branches are closed or all formulas are checked. A completed tree is called *open* if there is a branch in it, which has not been closed. A closed tree for the negation of a formula, means that the formula is a tautology. For the convenience of the reader, the sign "x" is used below branches closed by a propositional contradiction, i.e. closed by a propositional formula and its negation. The sign "#" is used below branches closed by contradiction of imperative formulas. The signs "x" and "#"are not regarded part of the actual tree.

Rule for Corresponding Proposition

For any atomic imperative X or negated atomic imperative ¬X in a branch and their corresponding propositions x and ¬x, we may add x∨¬x to that branch. Compared to the semantics, this rule reflects that the corresponding proposition is always given a truth value in a given valuation.

Fulfilment Rules

B	¬B
b	¬b
――――	――――
B:f	¬B:f

That is, we show the fulfilment values an imperative get underneath the branch, in the format B:fulfilment value. It can be read, B (or ¬B) is fulfilled in the branch above. In these two rules and the two following the order of the formulas in a branch may be reversed (e.g. so B occurs after b)

Violation Rules

B	¬B
¬b	b
――――	――――
B:v	¬B:v

This can be read B(or ¬B) is violated in the branch above.

Paradoxes

It is now time to check LM against some of the paradoxes of standard deontic logic.

Conditional Paradoxes

Arthur Prior´s Paradox of Derived Obligation

Prior´s paradox contains no permissives. It can therefore be represented LM. The conclusion of the worst half of Prior´s paradox was:

…the doing of what is forbidden commits us to the doing of anything whatsoever[219]

The formalisation in this system is:

12) $\neg Q \rightarrow (q \rightarrow S)$

To facilitate understanding I interpret the atomic sentences informally.

Q: Hit Andrea!

q: You Hit Andrea.

S: Kick Andrea!

The translation of the whole sentence then goes: "Don´t hit Andrea, but if you do hit her, then kick her too!". If there is an open branch it can be turned into a counter example to the logical validity of the original formula.

Tree 12.1)

1.	$\neg(\neg Q \rightarrow (q \rightarrow S))$	Negated Test Formula.
2.	$\neg Q$	From 1 by $\neg\rightarrow$- rule.
3.	$\neg(q \rightarrow S)$	From 1 by $\neg\rightarrow$- rule.
4.	q	From 3 by $\neg\rightarrow$- rule.
5.	$\neg S$	From 3 by $\neg\rightarrow$- rule.
	$\neg Q$:v	From 2 and 4, v-rule.

The numbers of lines and the explanations on the left are just there to help the reader. Line 2 is obtained from line 1 by applying the rule for negated conditionals to 1. The last line does not get a number because it is written underneath the tree. The tree does not close, and can be turned into a counterexample to Prior´s "paradox". This would be a valuation where, you are not allowed to hit Andrea($\neg Q$:i), you actually do hit her(q:t), but you are not allowed to kick her($\neg S$:i). In this valuation the mandate $\neg Q$ is violated because Q has anti-designated value and q designated. Your violation of one order, does not make anything legal in LM.

[219] A.N. Prior, "The Paradoxes of Derived Obligation", *Mind,* 1954, p.64.

Chisholm´s Contrary-to-Duty Imperatives

Consider the following.

13) Go help your Neighbours!

Tell them you are coming if and only if you go help them.

You are not going to help them. [220]

Above we found no adequate way of formalising these using SDL. Now we use LM.

Q: Go help your neighbours!

q: you go help your neighbours

T: Tell your neighbours you are coming!

Tree 13.1)

Q Premise
$T \leftrightarrow q$ Premise
$\neg q$ Premise

T	$\neg T$	
q	$\neg q$	
x		
	t	$\neg t$
	Q:v	Q:v
	$\neg T$:v	T: f

There are two open branches. In the left open branch, Q and ¬T are both violated. This is a valuation where you call them and don´t go. This is at least as bad as simply not going. It is however just as logically possible, but this possibility is not even shown by SDL. In a valuation corresponding to the right branch the primary obligation Q is violated but the secondary obligation ¬T is fulfilled. I have skipped a step ($t \vee \neg t$) and simply added branches for each disjunct to the open branches. It may also be seen that the premises *implies* the conclusion ¬T. This is a moral cue for the acting addressee. To see this add ¬¬T to the open branches and the tree will close.

Gentle Murder Paradox

We saw that in SDL the following set of sentences entail a contradiction.

15) Do not murder Jones!

If you do murder Jones, do it gently!

You murder Jones.

You murder Jones gently *entails* you murder Jones.

We formalise the atomic sentences

Q: Murder Jones

q:You murder Jones.

R: Murder Jones Gently!

r: you murder Jones gently.

15.1)

```
                ¬Q
                q→R
                r→q
                r∨¬r
                  q
              (        )
            ¬q           R
             x       (       )
                  ¬r          q
                 (    )     (    )
               ¬r     r  ¬r      r
               R:v    x  R:v    Q:v
               Q:v       Q:v    R:f
```

Naturally all branches involves ¬Q being violated because "you murder Jones" is taken to be true. On the other hand in the right-most branch, the one where you murder him, but murder him gently, the contrary-to-duty imperative is fulfilled, whereas it is violated in the two remaining branches, which can be turned into valuations where you do not murder him gently.[221] I have not added any pragmatic rules to the effect that fulfilled branches are preferable, and that if all branches contain the same violation, the one with no additional violation is preferable. Often this would be the case.

Ross´ Conditional Fallacy

Ross Conditional Fallacy is that the following is a valid inference in a logic of fulfilment, that is a logic which defines validity in terms of fulfilment of mandates.

16) If you write a paper, turn the paper in!
Write a paper!
____
Turn the paper in!

This is a fallacy, as Ross, Castañeda, and others have noted. The issuing of a mandate does not ensure that it will be fulfilled. I could be told to write a paper, but if I do not do it, it should not be

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[220] This is the imperative version of R.M. Chisholm´s original example, 1964, pp. 34-35.

[221] Nothing essential would change with regard to this paradox if we made rules for strict implication.

obliged to turn it in just from the premises above, because the condition for the implication does not obtain. We formalise

Q: Write a paper.

q: you write a paper.

R: Turn the paper in.

16.1)

$q \rightarrow R$ Premise

Q Premise

$\neg R$ Negated conclusion

$\neg q$ R

Q:v #

In the only open branch, the branch on the left, the mandate Q is violated, which represents a violation of Q. Since the tree remains open the conclusion is not logically implied by the premises.

Ross´ Disjunctive paradox

The question of fulfilment and violation is particularly important when considering Ross´ disjunctive paradox. In the imperative version it can only take the following form.

17) Send the letter!

Send the letter or burn it!

This inference is valid in LM. Let us draw the tree for the premise and the conclusion, without negating the conclusion.

Q: Send the letter!

R: Burn the letter!

17.1) Q

$Q \vee R$

Q R

This tree simply depicts that we are working with a non-exclusive disjunction for mandates and propositions and the remarks about this for SDL carries over to this context(see above). It is however understood that these two mandates cannot both be fulfilled at the same time. Since we deal with settled mandates one of these two mandates *is* violated in a valuation. We add the premise

¬q∨¬r. What this means, is that it is either true that you don´t send the letter or that you don´t burn it or that you do neither, or equivalently, that you do not both burn the letter and send it.
We now get a new and more revealing tree.

17.2)

```
                     Q
                    Q∨R
                   ¬q∨¬r
                   ¬q∨q
            /                 \
          Q                     R
       /      \             /       \
     ¬q        ¬r         ¬q         ¬r
    /   \     /   \      /   \      /   \
  ¬q     q  ¬q     q   ¬q     q   ¬q     q
  Q:v    x  Q:v   Q:f  Q:v    x   Q:v   Q:f
                                  R:v   R:v
```

It is seen that the only branch without any violated mandates is the fourth branch from the left. In this branch Q is fulfilled, and r is not true(you don´t burn the letter), but R is also not required in this branch, so therefore R is not violated. You therefore do not violate any mandates by sending the letter. We can simplify the discussion through assuming a stronger premise than (¬q∨¬r)by adding (q∨r), that is you will either send the letter or burn it. We then have (q→¬ r) and (¬r→q), which combined are equivalent to (q↔¬r).In situations where you have to choose one of the alternatives, this is the only right formalisation, but here I am only trying to simplify the discussion.

17.3)

```
            Q
            Q∨R
            q↔¬r
         /        \
        q          ¬q
       ¬r          ¬¬r
     /    \         r
    Q      R      /    \
   Q:f    Q:v    Q      R
          R:v   Q:v    Q:v
                R:f    R:f
```

In this tree the first branch from the left is the only one with no violated mandates. In this branch you send the letter and you do not burn it. We have to distinguish logical contradictions from practical situations where a particular mandate will have to be fulfilled not to violate any.

The discussion of the “paradoxes” above, which turned out not to be paradoxical or even strange in this system, lead us to the more important question of moral dilemmas. In the next section I will apply LM to this question.

Moral Dilemmas

A moral dilemma may be described as a situation where an agent is required to perform a set of actions, which are mutually exclusive.[222] Often only two actions are considered. As an example take a doctor who has two patients. He is required to save both but he only has time for saving one of them. Using SDL and alethic modal logic we get what could be called *the standard logical definition of a moral dilemma:*

18) $(\mathbf{O}A\wedge\mathbf{O}B) \wedge \neg\Diamond(A\wedge B)$[223]

A and B are both obligatory but they are also logically incompatible. As a special case we have

19) $\mathbf{O}A\wedge\mathbf{O}\neg A$[224]

which we distinguish and call a *normative contradiction*.[225]

The possibility of moral dilemmas has been denied from a logical standpoint. [226]Indeed the standard definition is contradictory if we accept the axiom $\mathbf{O}P\rightarrow\Diamond P$, that what is obligatory must be possible.[227] As we have seen in a previous chapter however, so are other sentences, which seem intuitively okay. At the other extreme, philosophers have also denied the applicability of logic to ethics at all and to moral dilemmas in particular. [228]

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[222]Cf. (Horty, 2003), p. 1:" *a normative conflict* is a situation in which an agent ought to perform an action A and also ought to perform an action B, but in which it is impossible for the agent to perform both A and B." Also (Hamblin,1987), p. 168, where he defines *quandaries* as:"situations in which an agent cannot act without infringing at least one of a given set of imperatives".

[223] E.g. (De Haan, 2001), p. 272. Also (Priest,2002), p.11.

[224] A special case because we always have $\neg\Diamond(A\wedge\neg A)$.

[225] Cf. (Morscher, 2002), p. 485, where he argues that since $(\mathbf{O}A\wedge\mathbf{O}\neg A)$ is a contradiction in SDL, it cannot be discussed usefully in this framework. The standard definition should be adopted.

[226] E.g. (De Haan, 2001)

[227] From $OA\wedge OB$ follows $\mathbf{O}(A\wedge B)$ in K(and thus D). From this and the Axiom $\mathbf{O}A\rightarrow\Diamond A$ follows $\Diamond(A\wedge B)$ contradicting $\neg\Diamond(A\wedge B)$. $\mathbf{O}A\rightarrow\Diamond A$ is justified if perfect worlds are possible worlds.

[228] E.g. (Cohen, 2003), p.xvii: Never try to break the issues down into their "logical form", as a friend of mine tried to do. He went quite mad of course, and is now reduced to teaching "corporate governance" poor fellow.

An interesting case is what has been called *conditional conflicts*, where the antecedent of two conditionals being true entails a normative contradiction: $C \rightarrow \mathbf{O}A$ and $C \rightarrow \mathbf{O}\neg A$(or $C \rightarrow \neg\mathbf{O}A$) and C.[229] In this case the contradiction only arises under certain conditions.
The status of the possibility diamond in the standard definition poses a problem to some. C.L. Hamblin writes about moral dilemmas(quandaries):
...we first run into the question whether possible, *as it occurs explicitly or implicitly in them, means* logically possible, *or* physically possible, or *actively possible* or *choosable*.[230]
In some cases there will be a *prima facie* moral dilemma though no *actual* moral dilemmas. Of the conflicting obligations one override the others. This is not always a solution. I may be required not to park my car in front of the drug store. Yet I have to do it in order to get into the store right before it closes. I get my medicine, thereby fulfilling one obligation, but I also get my parking ticket as a consequence of my other broken obligation. Also in the context of LM violated mandates must be distinguished from overridden mandates. The *prima facie*/*actual* obligations distinction works in the latter case, not in the former. Moral dilemmas are cases where one obligation *cannot* be said to override the others, otherwise there would not be any dilemma. That one has to choose one option, thereby violating at least one mandate is a different matter.
Let us look at an example of a moral dilemma and extract a logical definition of moral dilemmas in LM from the example

Cave Dilemma

A group of scientists are exploring a long and very narrow cave full of snakes as the roof falls on one of the members, Tom. Tom is now laying in the middle of the cave under a big pile of rock, which has fallen on him. Tom is alive but it is not possible to get him out from the inside. The other adventurers in the party are stuck behind the rock, but they have some explosives. The explosives would most likely remove the obstacle but the explosion would also kill the man stuck under the rock. If they don´t do anything the rest of the party will die from lack of oxygen, and hundreds of deadly snakes very soon. Tom has his head out on the other side, he breathes fresh air, and the snakes are only in the cave, so a rescue team might save him in the next day or so.
Now, to simplify things let us assume that a leader will make the decision. He is under the following orders.

[229] Cf. Nicholas Asher and Daniel Bonevac, "Common Sense Obligation", *Defeasible Deontic Logic*, p. 161.
[230] C.L. Hamblin, *Imperatives*, p. 181.

Do not kill Tom!

Save the rest of the party!

T: Kill Tom!

S: Save the rest of the party.

t: You kill Tom.

s: You save some members of the party.

The leader is under the obligation $(\neg T \wedge S)$. At the same time, it seems that the situation created in the thought experiment implies that"You either do not save the party or you kill Tom". We would formalise this $(t \vee \neg s)$. Adding this up we have $(\neg T \wedge S) \wedge (t \vee \neg s)$. More generally for Mandates A and B and their corresponding propositions a and b a *weak moral dilemma* is defined:

20) $(A \wedge B) \wedge (\neg a \vee \neg b)$

The weak moral dilemma can be strengthened to a *strong moral dilemma* by adding the premise $(a \vee b)$. The conjunction of this with $(\neg a \vee \neg b)$ is equivalent to $(\neg a \leftrightarrow b)$. What this means is that you fulfil one mandate and at the same time you violate the other. The definition is

21) $(A \wedge B) \wedge (\neg a \leftrightarrow b)$

In the example we are considering adopting the strong definition gives us $(s \leftrightarrow t)$ which we translate: "you save the party if and only if you kill Tom".

Now we can draw the tree.

22)

$$
\begin{array}{ccc}
 & \neg T \wedge S & \text{Premise} \\
 & s \leftrightarrow t & \text{Premise} \\
 & \neg T & \\
 & S & \\
\swarrow & & \searrow \\
s & & \neg s \\
t & & \neg t \\
\neg T{:}v & & \neg T{:}f \\
S{:}\ f & & S{:}v
\end{array}
$$

Each branch represents one "horn" of the dilemma. There are two logical possibilities and each of them involves a mandate being violated. In the left branch you save the party but you kill Tom. In the right branch you do not kill Tom but you do not save the party. If anybody should suspect this use of trees the following table is a "cut back" logical value table of the possible valuations of the weak definition. The last column has the fulfillment values of the mandates in the valuation.

Table 20.1)

A	B	a	b	$A \wedge B$	$\neg a \vee \neg b$	
i	i	1	0	i	1	B:v A:f
i	i	0	1	i	1	A:v B:f
i	i	0	0	i	1	A:vB:v

For $A \wedge B$ to have designated value both A and B need to have designated value, which is depicted in the table. For $\neg a \vee \neg b$ to have designated value one of a or b need to be false. For the strong definition simply delete the bottom row of the table, that is, rule out the valuation where a and b are both false. The table gives the same information as the open tree.

The right formalisation depends on the particular dilemma. Take for instance Sartre´s dilemma of a man who feels obliged to go to war to save France and to stay in town to take care of his sick mother. He might choose to violate both of these mandates if he could find some better alternative. For instance, he could go to help in a sanatorium for soldiers returning from battle and take his mother with him to take care of her. To leave this possibility open we would have to use $(Q \wedge R) \wedge (\neg q \vee \neg r)$ as the right formalisation.

It may be objected against my definitions that they are both much too weak. According to these any situation where you have to violate one of your obligations entails a moral dilemma. This is true. I could adopt the stronger definition $(A \wedge B) \wedge \neg \Diamond (a \wedge b)$ but it seems pointless for this simple logic. Whether something is "possible" for somebody is also often a practical decision when dealing with moral dilemmas. It seems like we need to adopt some meta-principle like, "it is not your own fault that $\neg a \vee \neg b$" for it to be a true moral dilemma. I simply take a moral dilemma to be a set of mandates and propositions which are not contradictory, but which are such that one of the mandates is violated in any valuation of the whole set. As I said above I now return briefly to Mally´s *Grundsatz 1*. In LM it is $((A \rightarrow B) \wedge (b \rightarrow c)) \rightarrow (A \rightarrow C)$. Although this is not valid, its negation entails a moral dilemma violating C or B in any open branch. Thus LM distinguishes practical dilemmas(which may happen in the world) from logical contradictions.

Contradictions of Mandates

Moral dilemmas must be distinguished from contradictions, which are logically impossible in LM.The simplest form for a mandate Q is $(Q \wedge \neg Q)$. An example is, "Open the door, but don´t open it!". It is probably rare for a moral or legal system to contain mandates and their negations. It is however more likely, at least in daily reasoning, that we have *conditional imperative contradictions.*

These arise when a certain set of imperative implications and a certain set of propositions (conditions) give rise to an imperative contradiction. 23) is an example.

23) If it is his birthday, give him a present!

Give him a present only if you have a dollar or more for a gift!

It is his birthday.

You don´t have a dollar.

Let us formalise the atomic sentences:

Q: give him a present!

r: it is his birthday.

s: you have a dollar or more for a gift.

23.1)

$r \rightarrow Q$ Premise
$Q \rightarrow s$ Premise
r Premise
$\neg s$ Premise

$\neg r$ Q

$\neg Q$ s $\neg Q$ s
x x # x

The tree closes, which means that the premises are inconsistent. The imperative premises give rise to an imperative contradiction under the conditions of the last two premises. As a way out of the contradiction exceptions can be defined on the sentential level. Change the first premise to $r \rightarrow Q \vee \neg s$:"If it is his birthday buy him a present, unless you don´t even have a dollar for a gift."

23.2)

$r \rightarrow Q \vee \neg s$ Premise
$Q \rightarrow s$ Premise
r Premise
$\neg s$ Premise

$r \rightarrow Q$ $\neg s$

$\neg r$ Q $\neg Q$ s
x x

$\neg Q$ s
x

This new tree has an open branch, the second from the right. In this branch you basically required not to buy him a present, because the condition of you not even having a dollar obtains.

Limitations of LM and Further Research

There are some clear limitations of LM. Some have already been pointed out. The system is simple and almost as easy to use as classical propositional logic but its simplicity is also a limitation. The most serious flaw that I see at this point is its failure to express permissions. Another obvious limitation is the lack of quantificational resources. This may be remedied by constructing a first-order semantics for LM. The considerations about first-order standard deontic logic in the historical chapter suggest that this is not an entirely simple matter. On the other hand the rewards would be great, especially when formalizing legal or moral systems applying to several persons or objects. Other possible extensions could be dynamic or temporal resources to go from one to the following normative state with different sets of normatives in force.

On the positive side the system avoids any of the deontic paradoxes known to me, without any artificiality. It is furthermore, as far as I know, the first system to successfully build in and separate both the features of normatives being in force and the normatives being fulfilled or violated into the formal semantics in a way so that the logical element is not based on fulfilment or violation but only force. At the same time it is always clear in which valuation a mandate is fulfilled and which it is violated[231] This is an original contribution, as is the theory of meaning for imperatives. It is also a plus that the concept of "truth" is not in any way attempted extended to mandates, even though logical features such as validity and contradiction are extended in what seems to me a natural way. What I like the best myself about the thesis is that it opens a way to use other semantic concepts than truth as basic in normative reasoning. We have seen in the historical chapter (e.g. (Rescher, 1966)) that this is not new but also that much mainstream logic does not do this. For alethic cases there is many-valued logics and so on. But for normatives I think this road is indeed very profitable and the analytic potentials of a full-fledged system seem very promising. Especially first-order normative logic, higher-order normative logic and defeasible normative logic may benefit from these ways of thinking.

Soundness of LM is proved in appendix 1.

DEONTIC LOGICS AND IMPERATIVE LOGICS

- A HISTORICAL OVERVIEW OF NORMATIVE LOGICS FROM ERNST MALLY TO DEFEASIBLE DEONTIC LOGIC AND A NEW SYSTEM

MARTIN MOSE BENTZEN

Summary

This final thesis (Danish: *speciale*) deals with logics of obligations, permissions and prohibitions.

Part I.

The first part of the thesis consists of a historical survey of deontic and imperative logics from 1926 to the present. The author presents the main developments of the field, as represented by the work of Mally, Jørgensen, Ross, Von Wright, Prior, Chisholm, Rescher, Hintikka, Castañeda, Forrester, and others. The logics considered are logics of commands, temporal deontic logics, defeasible deontic logics and others. A thorough exposition is given of standard deontic logic and quantified first-order standard deontic logic employing relational (possible world) semantics and analytic tableau. Both technical and philosophical issues are taken into account. Among these the deontic paradoxes, for example Ross´ paradox and the gentle murder paradox, problems of formalisation especially regarding conditional obligations, and the problem of choosing a suitable formal semantics for first-order deontic reasoning.

Part II

In the second part of the thesis the author presents a new system. This consists of two sections.

(A) A *theory of meaning for imperatives*. A theory of meaning diagnoses the basic semantic features of a fragment of language to be analysed and treats them philosophically. The basic intuition underlying the theory of meaning for imperatives presented here is that standard theories are heavily biased towards employing *truth* as the only important semantic concept. Instead the author argues that imperatives are *in force* or not *in force,* and that they can be either *fulfilled* or *violated*. As a consequence of this imperative force values and fulfilment values are given to imperatives in the formal semantics of the logic developed in (B). Context-free imperatives are singled out, serving as a justification for *mandates*,

imperative semantic counterparts to propositions.

(B) A *logic of mandates.* The logic of mandates is a logic of imperatives especially relevant for situations of decision making where an answer in form of a Do! or a Don´t! is required and an answer in form of a "you decide" is not appropriate. The logic shows, which mandates are fulfilled and violated. Conditional imperatives have traditionally caused problems, but these are also handled well by this logic. The logic is given a formal semantic exposition and then supplied with an analytic tableau proof-procedure. The logic is then applied to the deontic paradoxes and to everyday normative reasoning including moral dilemmas and conditional dilemmas. Limitations and possibilities for further research are considered.

This thesis is written in English.

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Appendix 1: Soundness of LM

The following shows that the basic rules of LM are sound, that any theorem proven is valid. We first extend the semantic concept *satisfiable* to branches of trees and to trees.
A set of formulas is satisfiable if there is a valuation giving all imperative formulas the value i and all propositional formula the value 1. A branch in a tree is satisfiable if the set of formulas consisting of the terms(formulas) on the branch is satisfiable. A tree is satisfiable if one of its branches are. The origin of a tree is the negation of the formula to be proven. From:
1) For all x and all y, if x is a tree and y is the origin of x, then if y is satisfiable then x is satisfiable.
and
2) For all x, if x is a tree and x is closed, then x is not satisfiable.
It follows that
3) For all x and all y, if x is a tree and x is closed and y is the origin of x, then y is not satisfiable.
Which is equivalent to saying that the negation of the origin is valid, and hence that all theorems are valid and the system is sound.
It is 1) that requires the most proof. The proof leans on the following proposition.
4) Any application of a basic rule to a satisfiable tree yields a satisfiable tree.
It follows then by mathematical induction, that if the origin is satisfiable then any tree "made from it" is, because the origin is a tree and any rule applied to the origin yields a satisfiable tree, and any ruled applied to this yields a satisfiable tree and so on and so forth. Consider a satisfiable tree T and a valuation that gives all imperative formulas the value i, and all propositional formulas the value 1 in a branch B. Any rule is applied one of the formulas $\neg\neg X$, $\neg(XbY)$ or (XbY). For double negation there are two cases. a) X is an imperative formula or b)X is a propositional formula. a) The application of the double negation rule yields the extended set of formulas $\{T,X\}$. Since $\neg\neg X$ has the value i in the valuation $\neg X$ has the value ø and X has the value i, and so the extended set is satisfiable. Similarly for b).
For the remaining rules there are four cases to consider for each rule.a) X and Y are both propositional formulas.b) X and Y are both imperative formulas.c) X is an imperative formula and Y is a propositional formula.d) X is a propositional formula and Y is an imperative formula.
The following facts need to be established. 1) For each *non-branching* rule if the premise has designated value(i or ø) in a valuation then both conclusions do. An a) case if $X \wedge Y$ has the value i , then X and Y both have the value i in that valuation and so the extension of the set {T,X,Y) is satisfiable. 2) For each branching rule, if the premise has designated value in a valuation then one of its conclusions do. E.g. a c) case if $X \vee Y$ has the value i, then either X has the value i, or Y has the value 1. In the first case the set {T, X} is satisfiable in the second {T,Y} is. In either case the tree has a satisfiable branch and thus is a satisfiable tree. There are $8 \cdot 4=32$ binary cases to consider (with Smullyan´s unifying notation they can be limited, see (Smullyan, 1968)) and I have only considered 2. The reader should get the gist, though.

Since any closed branch contains an imperative formula and its negation or a propositional formula and its negation no closed branch is satisfiable, because according to the evaluation rule for negation no valuation can give designated value to X and $\neg X$. Since all branches are closed in a closed tree a closed tree is not satisfiable. From 1) and 2) it follows that 3), which can be checked with predicate logic. Hence the system LM is sound. Completeness could also be proved, extending the standard proofs of e.g. (Smullyan, 1968) as done above for soundness.